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EXPLORATION AND EVALUATION
OF
NEW GLASSES IN FIBER FORM

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ABSTRACT

The length effect on fiber strength was studied for E-glass and 994 glass fibers. These fibers were primarily from strands. Tensile test results confirmed the linear log strength-log length relationship previously established and that fiber strength increases with decreasing length. At short fiber lengths (below 0.5 cm), the strength-length curve showed a change in slope due to the presence of mixed flaw populations, which were indicated in bi-modal failure distributions in the vicinity of the slope change. This change in slope was not anticipated in an earlier concept developed for failure prediction on the basis of the length effect. A tentative revised model is suggested but must await confirmation by experiments with controlled fiber damage.

Some comparative tests on strand strength showed a strength reversal with decreasing test length. The reversal could be related to the effect of fiber collimation.

Comparison of fiber properties of the different glasses on the basis of strength, weight, Young's modulus, surface damage, and fiber length showed that 994 fibers are superior to E-glass by a factor of approximately 1.7 for longer fibers.

FOREWORD

This Final Report covers the work performed under contract NONR 3654(00)(X), under the direction of J. A. Kies, U. S. Naval Research Laboratory, Washington, D. C., from 15 November 1961 to 25 November 1962. Principal investigator on this program was G. K. Schmitz under the general guidance of John V. Long and A. G. Metcalfe. Other Solar personnel contributing to this work were: A. R. Stetson (glass technology), D. G. Clark and D. Roth (laboratory work), John A. Bauer (theoretical analysis), and R. M. Gardner (statistics).

The primary objectives of the program were the examination of the length effect on strength properties of glass fibers based on the statistical analysis reported in NRL 5098, and the development of a scheme to determine the strongest fiber for particular applications. A further objective was the systematic study of failure distributions at different strength levels, and the effect of distribution variations on the strength-length relationship.

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I. INTRODUCTION

The strength of glass fibers is considerably more than 500,000 psi, while the strength of glass fiber reinforced plastics may be one-fifth of that amount. Investigation to explain this difference in strength has revealed no satisfactory quantitative reason. Studies of glass-resin load transfer and the effect of incomplete resin impregnation to determine a physical explanation have been only moderately successful. The work discussed in this report was undertaken to study the behavior of glass fibers and strands from a statistical point of view with less regard to the exact physical form of the defects causing the loss of strength. It was believed that once the statistical information on flaw distributions is available concerning the elemental construction units of reinforced plastic, such as filaments and strands, predictions might be made for typical fabricated parts.

Glass fibers and glass-reinforced plastics appear to behave as brittle materials. One of the most characteristic traits of brittle materials is the effect of structural size on the fracture strength. To explain these effects, Griffith (Ref. 1) introduced the concept that glass contains pre-existing flaws, so that the fracture process is one of crack propagation rather than crack initiation. Weibull (Ref. 2) extended this concept by reasoning that the flaws are randomly distributed throughout the body and are of random severity. The "weakest-link" approach was adopted as a criterion of failure, or a brittle material fails when the stress at any one flaw becomes larger than the ability of the surrounding material to resist the local stresses. Applying the statistical laws of probability, Weibull assumed a reasonable distribution function and derived the expression:

$$S = 1 - \exp\left(-V \left(\frac{\sigma}{\sigma_o}\right)^m\right)$$

where S = probability of failure

σ = applied stress

σ_o = upper limiting strength

V = volume of material

m = index of relative number of flaws

This relationship is deficient because the applied stress must approach infinity as the probability of failure approaches unity (or certain failure). Kies (Ref. 3) suggested that a simple solution to this problem would be to choose the relationship

$$S = 1 - \exp\left(-V \left(\frac{\sigma - \sigma_u}{\sigma_o - \sigma}\right)^{\alpha}\right)$$

where σ is the applied stress

σ_o is the upper limiting strength

σ_u is the lower limiting strength (probably zero for glass)

α is the damage coefficient, equal to:

$$\alpha = - \frac{\log (V)}{\log \left(\frac{\sigma - \sigma_u}{\sigma_o - \sigma}\right)}$$

where the variables σ and V refer to the same probability of failure.

Figure 1 shows the behavior expected based on this model.

Freshly drawn glass monofilaments can lose strength in many ways. Exposure to certain atmospheres and mechanical handling are the principal means by which strength is lost. Both of these means indicate strongly that the loss of strength is related to events occurring on the surface. This contention is supported by observations (Ref. 4) from decoration techniques that typical flaw densities on glass fibers are 10^2 to 10^3 per cm^2, or flaws are 0.3 to 3 cm apart on fibers of 40×10^{-5} inch diameter. Accordingly, the approach taken in this work has been to assume that the statistical analysis should be on the basis of area rather than the more general volume basis developed earlier from the Kies and Weibull analyses. In the case of constant diameter fibers, a further generalization has been made in terms of length, L, rather than area.

Table I summarizes the three principal statistical analyses that have been applied to the data. Each system has its disadvantages. For example, the Gaussian and Weibull distributions do not indicate certainty of failure until the stress approaches infinity; and the Kies and Weibull distributions require the assumption or determination of an upper limiting strength. The second factor was the primary cause of the decision to examine the fundamental Gaussian distribution.

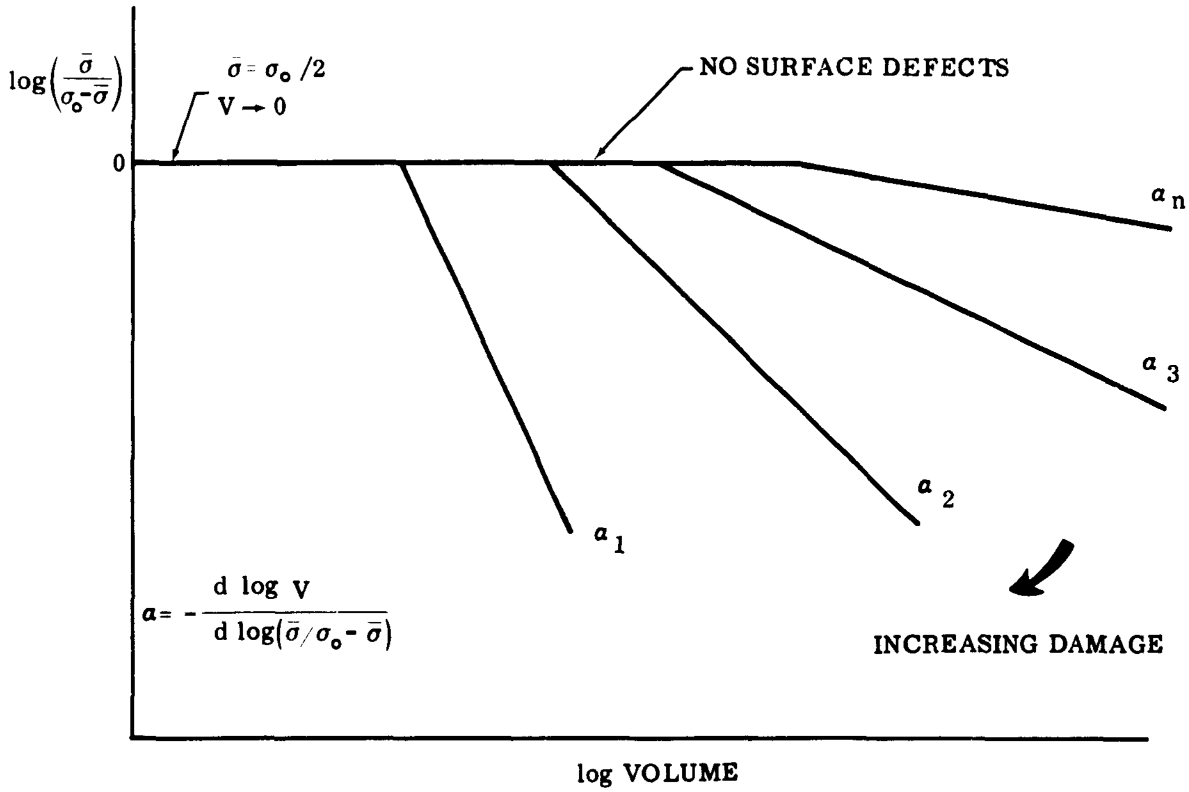

FIGURE 1. LENGTH EFFECT ON FIBER STRENGTH - SINGLE SURFACE FLAW POPULATIONS (KIES MODEL)

TABLE I

STATISTICAL ANALYSIS DATA

Parameter	Distribution		
	Gaussian	Weibull	Kies
Probability function	$S=1-\exp\left(-L\,\sigma^{2}\right)$	$S=1-\exp\left(-L\left(\frac{\sigma}{\sigma_0}\right)^{m}\right)$	$S=1-\exp\left(-L\left(\frac{\sigma}{\sigma_0-\sigma}\right)^{\alpha}\right)$
Strength function	σ	$\frac{\sigma}{\sigma_0}$	$\frac{\sigma}{\sigma_0-\sigma}$
Slope of log strength - log length curve	not applicable	m	α

Note: L = length

The experimental approach followed the general lines:

- Determine the strength of between 10 and 100 (generally 25 or 50) monofilaments for each gage length
- Analyze the data to determine the strength at 50 percent probability of failure, and to determine the best distribution or probability function to fit the data
- Using the logarithm of the strength function for the appropriate distribution, plot it against the logarithm of the length

The foregoing analysis permits information to be obtained on each of the following points:

- Statistics that fit best the distribution of strength values at any particular gage length
- Average strength (or strictly, strength at 50 percent probability of failure) for any fiber tested at each specific gage length
- The existence of an upper limiting strength for each fiber
- The existence of a lower limiting strength for each fiber
- The damage coefficient of each fiber
- New flaw distributions

II. EXPERIMENTAL WORK

The experimental program included tensile strength measurements on E-glass and 994 glass fibers, both monofilaments and separated from strands, and a limited amount of comparative strand testing. All fibers were Owens-Corning products. Fibers and strands investigated are listed in Table II. E-glass monofilaments were also drawn from a single bushing at Solar in preparation for a study of fibers with controlled surface damage. A trial drawing of 994 fibers was undertaken for the same purpose.

2.1 TENSILE TEST APPARATUS

The test apparatus and associated equipment were described in Progress Reports 2 and 3 (Ref. 5). The setup for short fiber tests is shown in Figure 2, including the bi-filar Gaertner micrometer-microscope for observation of gage length below 0.5 cm. Loads were read from a Daytronic 300B indicator and recorded on a Varian or Servo/Rite strip chart recorder. The load sensing unit was an LVDT, Schaevitz TD-3 with ± 350 grams range.

The gripping-wax carriers were made from car cigarette lighters (1/2-inch diameter), and were modified to permit testing of gage lengths as short as 0.025 cm. Red sealing wax was used throughout most of the tests; however, a wax with better gripping quality was used during the last test series. The melting temperature for successful fiber submergence was approximately 200 F. Wax temperature was kept as low as possible, especially in short fiber tests, to avoid the possibility of fiber annealing.

2.2 FIBER SLIPPAGE

The fiber slippage problem was studied critically because of its importance in short gage length tests. Two distinct types of fiber displacement were identified: First, slippage due to large scale separation of fiber-wax interface. Second, displacement due to yield of the wax surrounding the fiber.

TABLE II

SUMMARY OF FIBERS AND STRANDS INVESTIGATED
In Chronological Order of Testing

Glass	Finish	Designation	Length Range (cm)	Remarks
ECG 150 - 1/0	A-1100	Series I Series II	1.5 to 24 1.5 to 30	From three "milk bottle" spools. From one "milk bottle" spool.
E-glass and X-994	HTS	HTS (U-Frames)	3	Supplied by Owens-Corning, Hand carried.
X-994	195	Series I Series II	0.75 to 24 0.05 to 3	Strand, earlier production, date not known.
ECG 150 - 1/0	A-1100	Strand test	0.5 to 24	Same material as fiber test in Series II.
X-994	195	Strand test	1.0 to 34	Same material as above. Tested between Series I and II.
X-994	None	Virgin, (U-Frames)	0.5 to 7	Supplied by Owens-Corning. Hand carried.
X-994	None	Monofilament	0.25 to 12	Supplied by Owens-Corning. Wound interspaced on 6 inch cardboard drums. Shipped by air.
S-994 S-994	195 195	(July 1962) (July 1962) Long Fiber Survey	0.025 to 6 0.5	Supplied by Owens-Corning. Wound interspaced on 6 inch cardboard drums.
S-994 S-994	195 195	(9/19/62) (9/19/62) Undulated and straight	0.025 to 6 0.5	Supplied by Owens-Corning. Wound interspaced on 6 inch cardboard drums.
E-glass	None	Solar drawn	1.5	In preparation of E-glass study.
994	None	Solar drawn	1.5	Preliminary work based on published data.

FIGURE 2. TENSILE TESTER WITH REDUCTION GEAR FOR SHORT FIBER TESTING

Evaluation of the slippage error showed it to be small (above 0.1 cm) for fibers with finish. Uncoated fibers slipped much more severely, but length effect investigations on such fibers were at that time not extended below 0.25 cm.

To eliminate the effect of slip completely, an optical method was developed to observe if fracture occurred within the nominal gage length. A bi-filar micrometer-microscope was sighted on the fiber with the two filars spaced to the nominal gage length. One filar was referenced to a marking on the fiber, such as a characteristic spot on the finish. This reference was maintained throughout the test. To be valid, failure must occur within the bi-filar range. In essence, the fiber outside of this bi-filar range was regarded as an extension of the grips. Details of this method are given in Report 5 of Reference 5. Since slippage is generally undesirable, other waxes were evaluated.

2.3 DIAMETER MEASUREMENTS

A Leitz microscope, model Labolux 7.4.5.30P48/81, with Leitz filar eyepiece was used for diameter measurements. Light refraction effect was minimized by submerging the fibers in methyl-phenol-ether (Anisole) having an index of refraction of 1.518.

Because of the importance of the diameter measurements on strength computation, a standard technique for read-out was developed. The fiber was focused so that a thin white halo was observed adjacent to the dark-line fiber edges. The filars were set to barely bracket the fiber. The width of the filar was subtracted from the reading to compensate for the offset of the filar center lines. The average read-out error was ± 1 filar unit (one drum division) which is less than one percent of 112 units for a diameter of 40×10^{-5} inch.

Since it was frequently impossible to secure the fractured end of the fiber, the following method was employed to determine the fiber diameter: The two fiber ends extending beyond the gripping points were measured at two locations approximately 1/4 to 1/2 cm apart. Excess finish was stripped off before measurements. The fiber diameter was then determined from an average of four readings. Measurements of fractured ends, if available, proved the reliability of this method with one exception: The X-994 strand fibers showed diameter deviations as large as

10 percent (as determined from a survey of four 24-cm long fibers). In this case, the average cross-sectional area, derived from individual cross-sections of the 50 fibers tested for each gage length, was used for stress computation. This procedure was used in the X-994 Series I only. The shorter fibers in Series II had less diameter deviations. Strand sections used for these two series were 40 feet apart due to intermittently conducted strand tests which could also account for better diameter consistency.

2.4 FIBER SEPARATION

The majority of fibers tested were separated from strands. The degree of damage caused by this operation appeared difficult to assess. However, comparison of fiber and strand tests indicated that such damage might have been insignificant. The separation procedure was kept as constant as possible, and many trial separations were made prior to testing.

A typical separation of individual fibers for ten different test lengths proceeded as follows:

- A three-foot long strand was cut from the spool
- A partial strand was separated containing sufficient fibers for the sample size selected (Appendix B; Tables B-I, B-II). Usually, fibers from the edge of the strand were used.
- The partial strand was cut into approximately 4-inch sections; the sections were labelled consecutively in sequence of the planned test lengths. The remaining portion of the strand was marked for possible future use.
- Each 4-inch long section was subdivided into three groups and individual fibers were separated in rotating order.

To separate an individual fiber, one end of the bundle was taped to a support extending a few inches from a black background. Separation began at the lower end using a slender needle. Both fiber and remaining bundle were carefully taken between fingertips and slowly pulled apart with as little force as possible. Separation continued in a somewhat jerky motion depending on the amount of finish. This constituted the critical phase of the operation and great care was exercised not to exert large forces or create sharp bending at the points of temporary arrest. Observations during this procedure and microscopic examination led to the conclusion that the finish connecting one fiber with another forms occasional "bridges" which accounts for the degree of resistance to separation. In general, separation was fairly easy and uniform, and it is believed that finish-finish separation took place rather than finish-glass.

2.5 TEST PROCEDURE

The test apparatus was calibrated by deadweight in the direction of the load axis before and after each test series. Instrument sensitivity was selected for the highest anticipated load to fall in the upper scale range. The average combined calibration and read-out error in the middle and upper load range was ± 1.5 percent.

In preparing the tensile test, the individual fiber was mounted on a fork and then inserted into the molten wax. Care was taken to properly align the fiber in the load axis to avoid detrimental bending effects from wax at the fiber exit. Pretensioning of short fibers due to wax contraction during cooling was compensated by load reduction not exceeding zero load; i.e., no compressive bending took place.

The loading rate was adjusted to give fiber breaks within 20 to 60 seconds, corresponding to a strain rate of approximately 0.06 in/in./min. This rate was maintained throughout the tests. Fibers were observed during loading to locate the point of failure.[1] Fiber breakage occurred away from the wax in approximately 90 percent of the tests. Strength of fibers failing at the wax showed no tendency towards lower values which was attributed to careful alignment. These values were used except for the first test series (E-glass) where omission was a matter of principle. At gage lengths below 0.1 cm the rate of failure at the wax was high (20 to 50 percent) during the first short-fiber test series (X-994 Series II and S-994 (July 62)). These failures showed lower strength values which were omitted at the time.

Occasionally, fibers failed in the wax at 0.25 cm and below, in which case values were not used because of the unknown gage length. Some of the strength values were in the upper 10 percent strength range. This seemed to indicate (1) that the fiber-wax junction is not a critical point if bending due to improper alignment is avoided, and (2) that the annealing effect of the heated wax (approximately 200 F) is negligible.

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[1]Occasionally, circumferential breakage of finish was observed preceding fiber failure at that location.

Test data such as load, environment, diameter, and visual observation regarding fracture location were recorded on prepared sheets.

2.6 STRAND TESTING

A horizontal Hounsfield tensiometer, 'ype W, with a load range from 0 to 62 pounds was used for strand tests. The apparatus was equipped for direct load/crosshead movement plots.

Tensile test specimens were prepared in the following manner: Strands were pretensioned slightly (10 grams at each end). Specimen ends were sandwiched between resin impregnated cloth, one inch by one inch, and cured at room temperature for four days followed by one-half hour cure at 200 F. The same procedure was used for resin-impregnated specimens.

For testing, tab surfaces were sanded with emery cloth for firm gripping in self-gripping jaws. Slip tests, as well as frequent checks during actual testing, showed that slippage was constant, approximately 0.016 inch. Strain rates varied from 0.05 to 0.1 in./in./min. Failure normally occurred between 10 and 50 seconds.

2.7 FIBER DRAWING

E-glass fibers were drawn from Owens-Corning marbles through a single hole Pt-10Rh bushing. Preliminary drawing from a one-inch deep bushing was unsatisfactory. Improvements were achieved with a revised bushing, 1 inch square and 2-1/2 inches deep with a tapered bottom section and a wall thickness of 0.062 inch. The nozzle of 0.125-inch OD and 0.062-inch orifice extended 1/8 inch below the bottom. The bushing was imbedded in a K-28 insulation brick lined with zirconia and Y-310 binder material for airtight sealing. Temperatures were measured at three locations: in the melt, at the outer surface of the bottom of the bushing, and at the wall 1/2 inch above the bottom. The latter was used as reference temperature.

Fiber sampling began after five minutes of drawing by manually breaking the fiber between bushing and winding drum. The fiber was then mounted tension-free on a three inch wide U-frame having ten serrations approximately one inch apart.

Three to four samples of the desired diameter ($40 \pm 5 \times 10^{-5}$ inch) were drawn before marbles were added to the bushing in order to maintain a constant glass head. The frames were stored immediately in separate airtight compartments each containing silica-gel for humidity control. Testing started after the first U-frame had been prepared.

Trial drawing of fibers was made from the 994 glass formulation reported by Owens-Corning (Ref. 6). The glass was melted in a platinum crucible and monofilaments were drawn from the remelt at approximately 2850 F.

III. TEST RESULTS

3.1 DATA PROCESSING

The raw data from load and diameter read-out were processed by a computer program to obtain the following information:

- Average Values

  Strength, standard deviation, coefficient of variation, third and fourth moment, and diameter.

- Individual Values

  Load, diameter, cross-sectional areas, strength, Kies' strength parameter, and probability of failure.

3.2 DATA PRESENTATION

3.2.1 Tabulated Data

Appendix B contains a summary of the average values obtained from E-glass and 994 fiber tests. Also listed are the results from strand tests on both E-glass and X-994 glass. Individual fiber-strength data and fiber diameters are listed in Appendix C.

3.2.2 Graphs

Strength-Length Relationship

The strength-length relationship for both E-glass and 994 glass fibers is presented in the Weibull relation log (average strength) versus log length in Figures 3 and 4, and in the Kies relation, log (strength parameter) versus log length in Figures 5 and 6. Data from other sources are incorporated for comparison. Strand strength versus length is shown in Figure 12. For comparison, data from fibers separated from these strands are included.

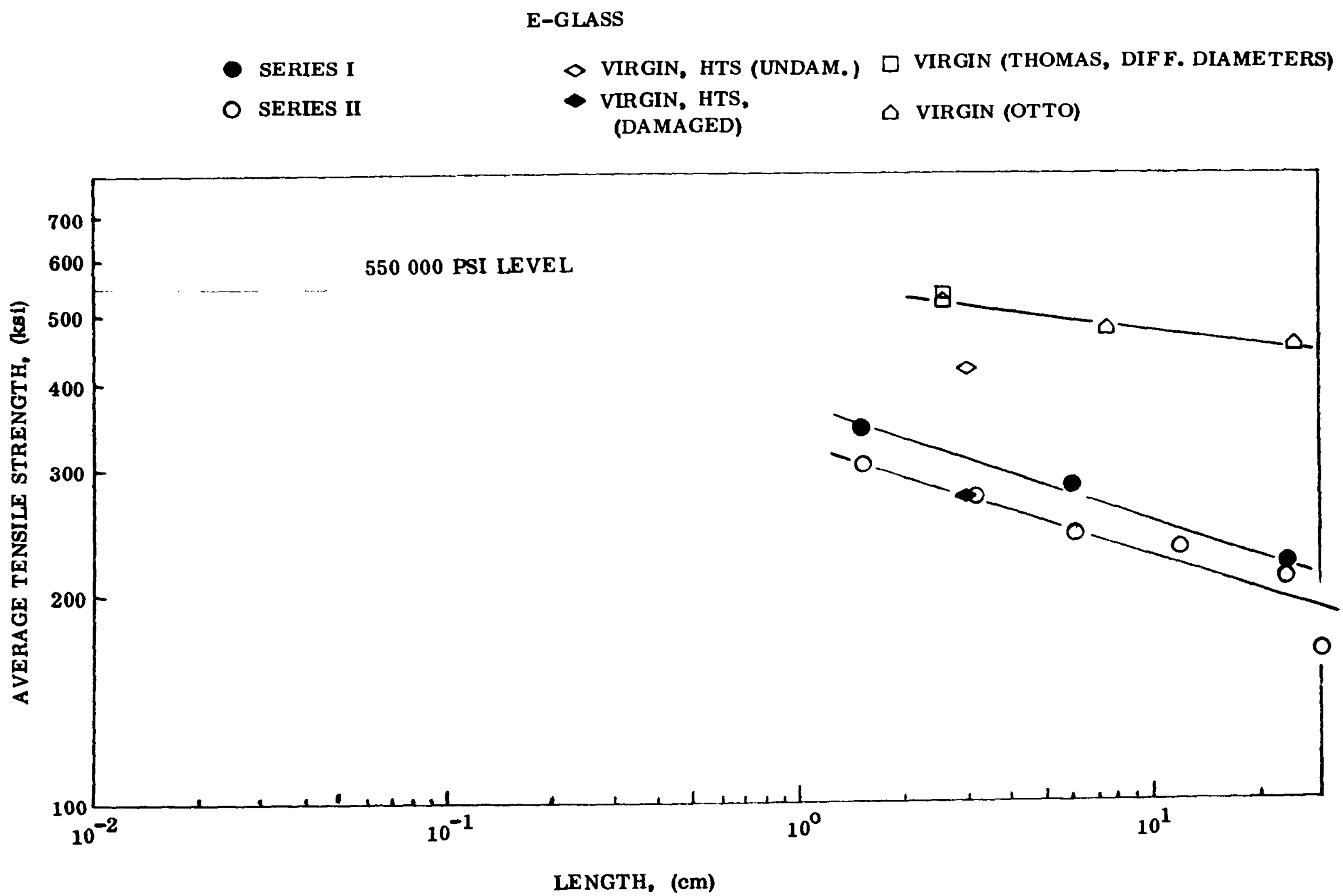

FIGURE 3. LENGTH EFFECT ON BREAKING STRENGTH OF E-GLASS FIBERS (WEIBULL RELATION)

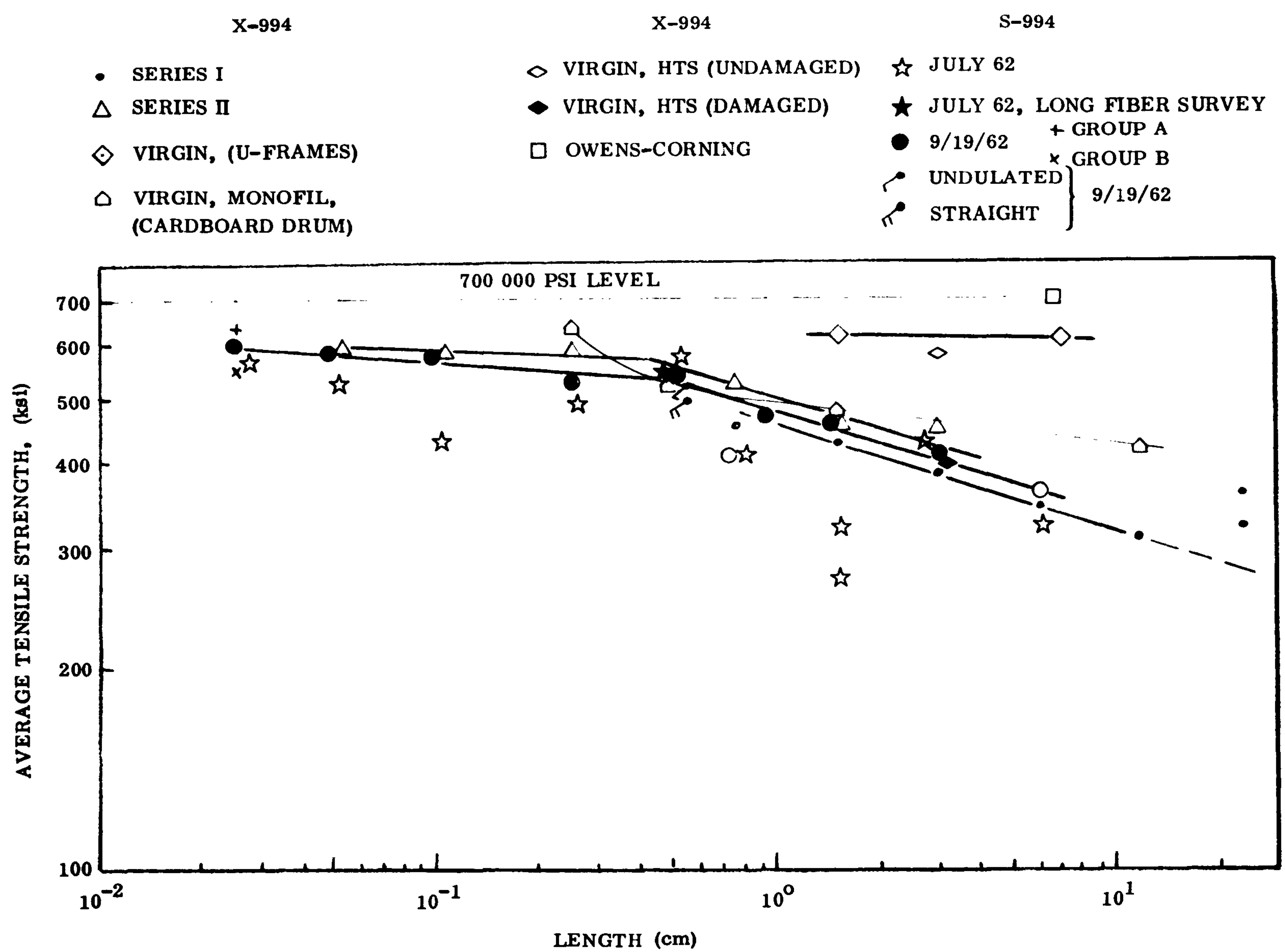

FIGURE 4. LENGTH EFFECT ON BREAKING STRENGTH OF 994 FIBERS (WEIBULL RELATION)

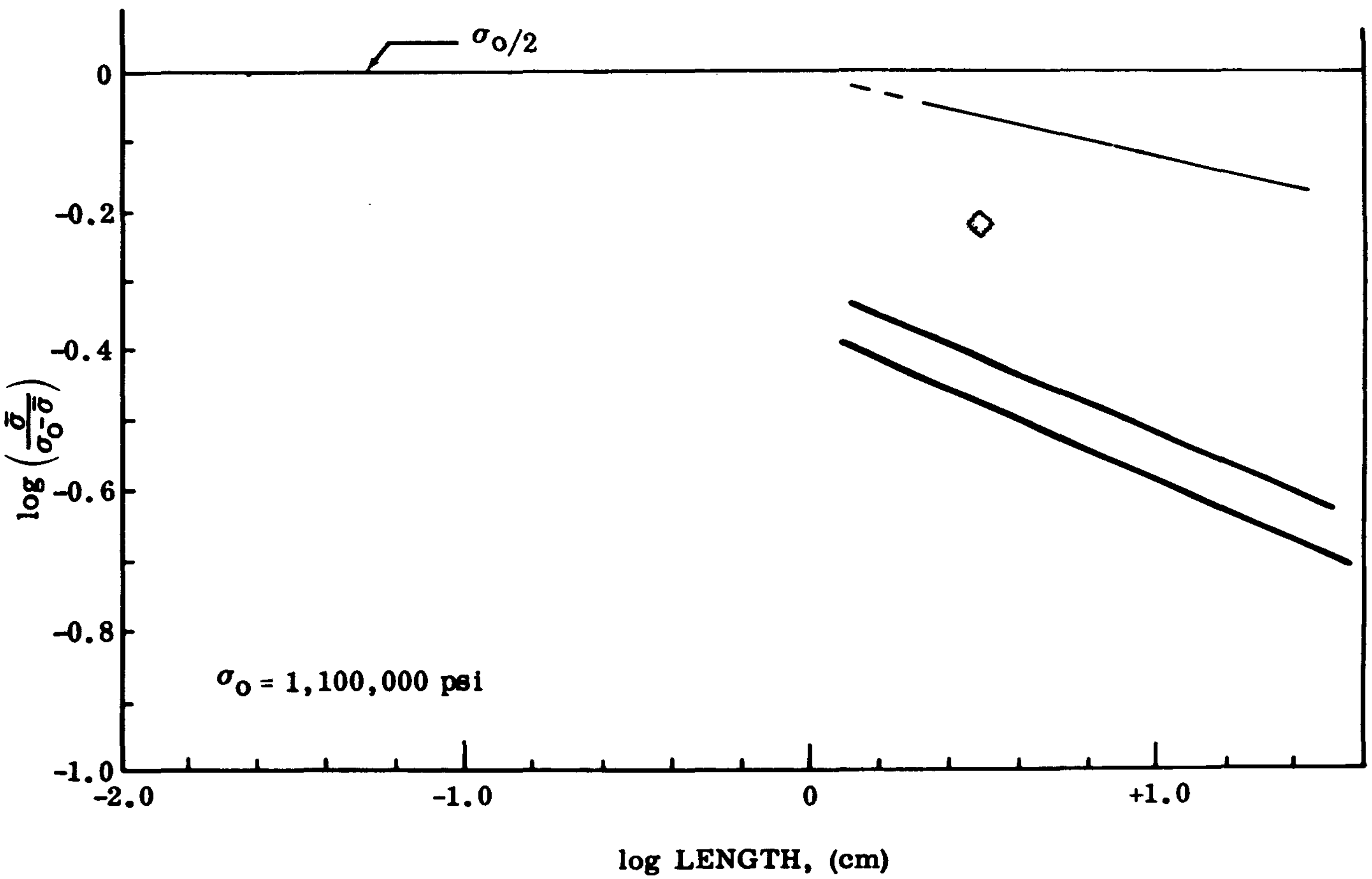

FIGURE 5. INTERPRETATION OF LENGTH EFFECT DATA - E-GLASS (KIES RELATION)

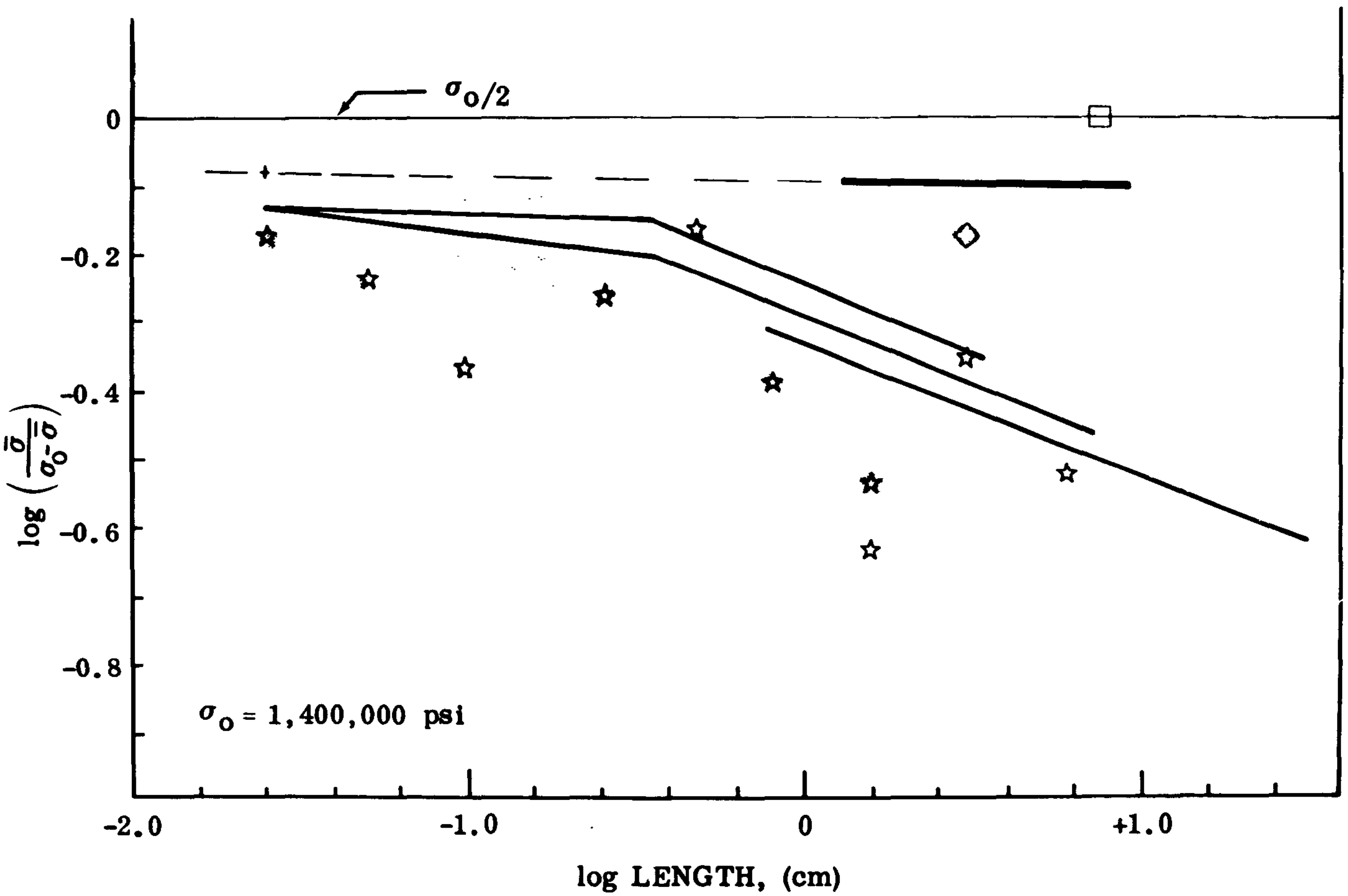

FIGURE 6. INTERPRETATION OF LENGTH EFFECT DATA - 994 GLASS (KIES RELATION)

Failure Distributions

Gaussian failure distributions were used throughout the progress reports and are not incorporated in this report except for the previously unpublished results on S-994 (9/19/62). These results are given in Appendix A.

In addition to the Gaussian failure distributions, Weibull's and Kies' distribution functions were used to investigate the occurrence of best straight line fit for the three distribution functions used in this program.

Diameter Distributions

Most of the diameter distributions were published in previous progress reports. Those not included, and the distributions from the S-994 (9/19/62) fibers, are presented in Appendix A.

IV. DISCUSSION

This investigation was initiated to study the behavior of glass fibers and strands from a statistical point of view, i.e., by means of failure distributions obtained from strength measurements at various gage lengths. Analysis of the distribution curves provides information that describes strength properties and behavior of fibers:

- The length effect on fiber strength
- The type of distribution typical for different fiber lengths
- Upper and lower strength limits
- Damage coefficient of fibers
- The relative strength of different glasses

In general, a plot of log (strength parameter) versus log length will give a straight line for the Gaussian, Kies, and Weibull distributions. Table I gives the values of the strength functions or parameters to be used for each distribution. The slopes of these plots are related to the density and severity[1] of surface defects. The Weibull slope, m, has been defined as the index of the relative number of flaws, and the Kies slope, α, has been defined as the damage coefficient. In both cases, the actual values of these slopes depend on the units adopted for length (or area), but the slopes provide a convenient basis for comparison of degrees of damage when self-consistent units are used.

The problem of the upper limiting strength, σ_0, in both the Kies and Weibull analyses has been mentioned earlier. A clear and forthright definition of σ_0 is not possible in terms of a physical model, such as theoretical strength calculated from bond energies. Indeed, such a definition is not to be expected because the probability functions have been advanced to provide a reasonable approximation of the behavior of the materials and are not based on the properties of

[1] The term severity describes the stress rise effect of a defect.

glasses. However, in the case of the Kies function, it has been shown (Ref. 7) that the upper limiting strength, σ_o, should approach twice the highest average strength of short test lengths. Using this result, it has been assumed that the upper limiting strength of E-glass is 1,100,000 psi and of 994 glass is 1,400,000 psi. In general, results are relatively insensitive to the values adopted.

A logical basis for the development of this discussion would be to examine the various probability functions first to determine the function that fits the experimental data best. Once this function has been determined, the log (strength parameter)-log length curves could be plotted using the appropriate strength parameter. However, the detection of a distinct break in the logarithmic (strength parameter)-length plot showed that more than one flaw distribution might be involved. The unknown effect of this break on the probability-strength plots made it mandatory to examine the strength versus length plots first.

4.1 THE LENGTH EFFECT

Figures 3 and 4 show plots of the experimental data on E and 994 glasses. The plots in the figures are against the average tensile strength on a logarithmic scale. This fits the requirements of the distributions of Gauss (log $\bar{\sigma}$) and Weibull (log $\bar{\sigma}/\sigma_o$, which represents a displacement of the stress axis by an amount -log $\bar{\sigma}_o$), and avoids the assumption of any value of the upper limiting strength, σ_o. Figures 5 and 6 show essentially the same data in a diagramatic form against the Kies strength parameter (log $\bar{\sigma}/\sigma_o - \bar{\sigma}$) using the values of the upper limiting strength given earlier.

Figures 3 and 5 show that the logarithmic strength versus length plot is linear for E-glass over the range of lengths studied. Included in these figures are data from Otto[1] and Thomas (Ref. 8) which show excellent agreement at 2.5 cm length. All of these results indicate higher strengths might be found at shorter lengths. Unfortunately, the test methods for shorter lengths were developed during the course of the study of 994 fibers and were not available at the time of examination of E-glass. In view of the planned study on E-glass in the virgin condition and after controlled amounts of damage, it was felt unjustified to repeat and extend these observations beyond that study.

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[1] Private communication

Figures 4 and 6 present the logarithmic strength versus length plots for 994 fibers. Tests with these fibers extended to considerably shorter lengths than the E-glass, and showed that a distinct change in the slope of the curve occurred at approximately 0.5 cm. Again a linear plot is obtained against both log $\bar{\sigma}$ and log $\bar{\sigma} / \sigma_0 - \bar{\sigma}$) for both the longer lengths (above 0.5 cm) and for shorter lengths. By analogy with the results for 994 fibers, a change in slope is postulated in the curve for E-glass and is shown diagramatically in Figure 5.

Examining first the results for long test lengths (all E-glass results and data above 0.5 cm for 994 glass), it is apparent that the slopes of the different curves increase as the strength decreases.[1] Compare the positions and slopes of lines in Figures 5 and 6. The highest strength (700,000 psi) for 994 glass reported by Owens-Corning is for a single length so that no comparison is possible. The virgin X-994 glass monofilaments, from U-frames examined by Solar, have average strengths over 600,000 psi and an extremely shallow slope. These fibers had been exposed for approximately two weeks in transit from Owens-Corning and this may account for the loss of strength (Ref. 5, Report 4). By comparison, Otto's results on virgin E-glass (Fig. 3) show a much steeper slope, similar to the Solar results for X-994 monofilaments supplied by Owens-Corning wound on a cardboard drum (Fig. 4). However, the sharp upturn in strength of the latter at 0.25 cm and other observations suggest that these results are peculiar to this particular sample of monofilament and should not be considered as representing the general behavior pattern. The bulk of the data on both E-glass and 994 glass fibers separated from strand show extremely consistent behavior and this behavior has been represented by the full lines in Figures 5 and 6. In each case where departures occur, explanations have been found such as the inconsistency of diameter on an early batch of S-994 production. The general conclusion can be drawn that as fibers are damaged to an increasing extent (as indicated by loss of strength) the slope of the logarithmic strength versus length curve increases. In line with this general conclusion, the scatter of low strength data of the early 994

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[1] Actually, slopes are negative and the increase is from a low negative value toward zero which follows from:

$- \frac{d \log (L)}{d \log (\text{strength parameter})}$. This should be kept in mind during further discussion of slopes.

fibers, specifically from the July 62 strand, can be explained as resulting from various levels of damage. The wide bands in Figure 6 represent two higher levels of damage and contain many of the scattered points.

With the short 994 fibers, the behavior pattern repeats itself, i.e., the strength again increases with shorter gage length. This increase is more distinct in one curve (S-994) than in the other (X-994) and signifies two different degrees of damage. Thus, the behavior characteristics of a material obeying the Gaussian, Weibull, or Kies distribution of flaws are exhibited over two different ranges of length. A sudden change in the damage coefficient appears to occur at some critical length. A logical explanation is that the behavior at long lengths is governed by a distribution of severe flaws separated by a distance averaging approximately that of the fiber length at the break. This length is 0.5 cm for fibers separated from strand. Cameron (Ref. 4) has shown that typical flaw densities on virgin glass detected by decoration techniques are 0.3 to 3 cm apart. If these flaws are regarded to be cracks having varying stress concentration effects, then the break in the logarithmic strength-length curve corresponds to a change in the strength-governing defects from surface cracks to some less severe defects. These less severe flaws, governing failure at short fiber lengths, are not so readily identified. The severity must be low because the strengths are high, and the flaw must be very closely spaced because of the small influence of length changes.

In conclusion, E-glass and 994 glasses obey typical relationships for brittle materials, such as:

$$S = 1 - \exp\left(-L f(\sigma)\right)$$

where S is the probability of failure for a fiber length L and f is a function of the failure stress. The slope of the log (strength parameter)-log length curve determines α . A change in slope of this curve for 994 glasses at 0.5 cm with observance of this relationship at shorter lengths was taken to indicate that a new distribution of flaws controls failure at short lengths. On the basis of Cameron's observations on crack flaw density on fibers, it is tentatively suggested that surface cracks govern the failure above 0.5 cm and a more subtle, unidentified flaw governs failure below this length.

4.2 FAILURE DISTRIBUTIONS

The observations of the length effect suggested that two flaw distributions were present on the 994 glass fibers separated from strands. One was tentatively identified as surface cracks on the basis of decoration technique results. These cracks have wide separation (~ 0.5 cm). The other was identified as a very mild flaw closely spaced, but the physical nature of the flaw was not identified. Further consideration showed that at and near the critical length, failure may be controlled by either flaw population. Hence, the probability plots might be expected to reveal the presence of this bi-modal distribution of flaws.

The existence of bi-modal distributions can be readily detected in failure probability plots by comparison with plots from single-mode distributions. Such a comparison is made in Figure 7. Single-mode distributions, indicated by straight line plots, exist at either end of the length range, 0.05 and 1.5 cm. Distributions in the transition zone of the change in slope are related to two types of flaws, A and B. With decreasing fiber length, the contribution of flaw type B increases until it governs the entire probability range.

Although the failure distributions indicate the existence of a mixed flaw population, no direct information about the nature of each type of defect can be extracted. One exception is the case of freshly drawn fibers where the slope change in the strength-length curve (at the highest average strength level) must be caused by the transition from surface to structural flaws, or more exactly, to weaknesses in structural bonds.[1] As far as the distributions shown in Figure 7 are concerned, the flaw type B might be a surface flaw.

A number of 994 failure distributions near the change in slope have been analyzed with respect to the flaw type ratio, A/B. Results are shown in Figure 8 in terms of percent population B versus log length. The considerable scatter is due to the small sample sizes involved and to the fact that this percentage ignores overlap of types A and B on either side of the break in the distribution curves, and the attendant possibility that a smooth transition exists rather than a sharp transition.

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[1] Experimental proof is expected from the forthcoming study on freshly drawn E-glass monofilaments.

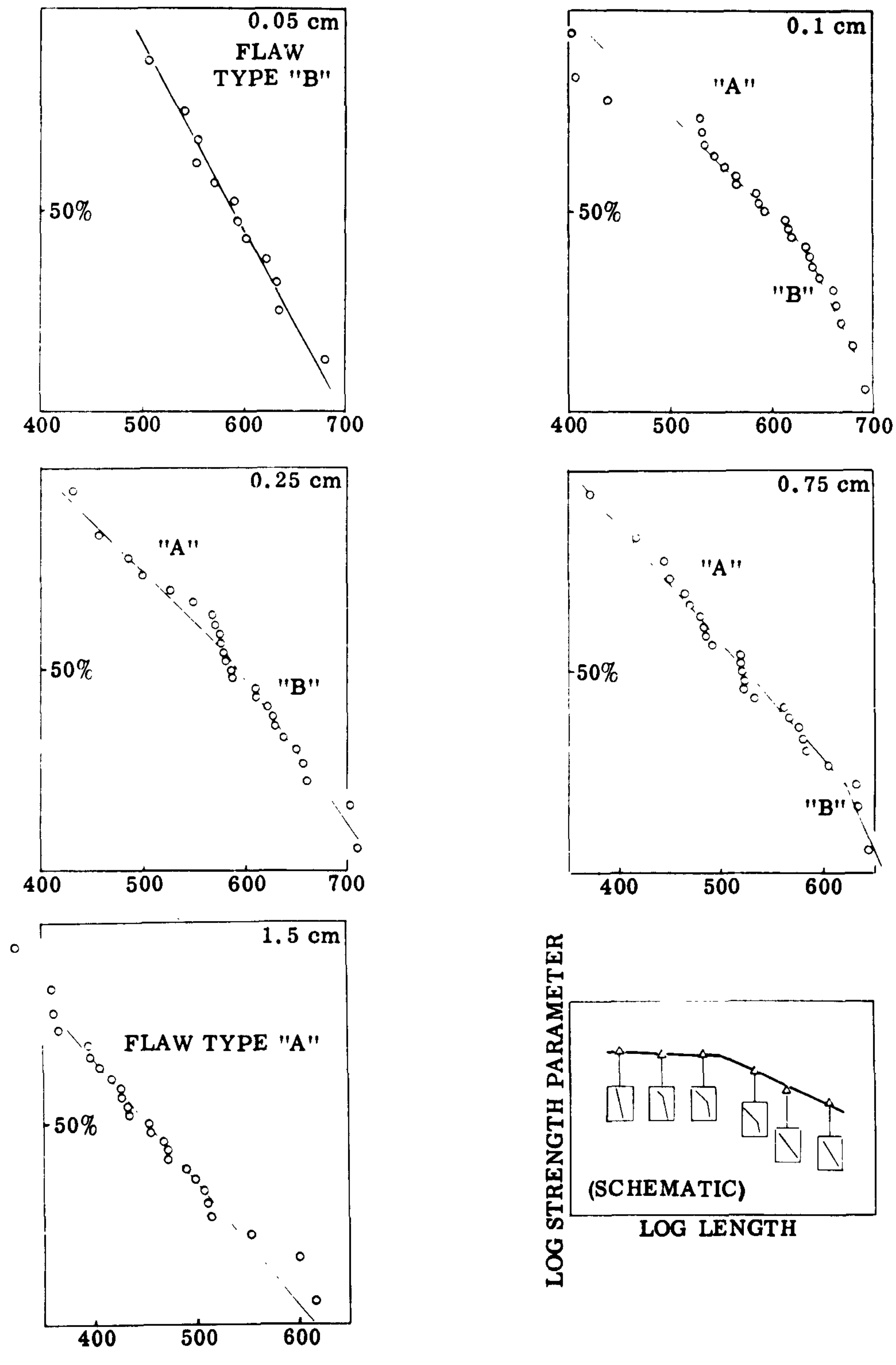

FIGURE 7. BI-MODAL FAILURE DISTRIBUTIONS AT THE STRENGTH-LENGTH SLOPE BREAK

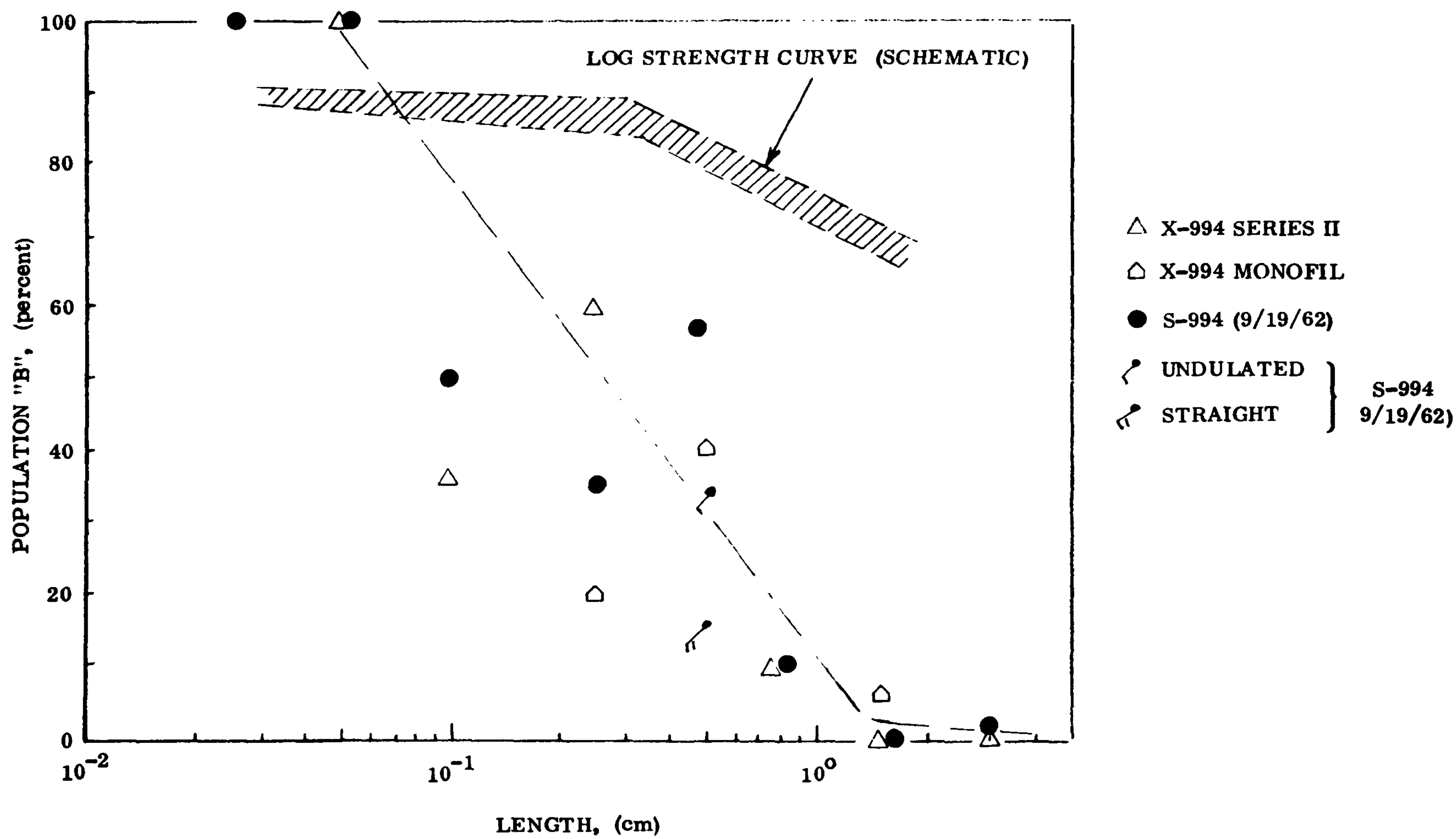

FIGURE 8. PERCENT POPULATION "B" VERSUS TEST LENGTH NEAR BREAK

A theoretical study has been made to express mathematically the bi-modal failure distributions. Concepts and preliminary mathematical descriptions are presented in Report 5 of Reference 5. An abstract and results of some additional work are given in Appendix D. The validity of these concepts will be tested by the programmed work with large sample sizes of Solar-drawn E-glass fibers subjected to controlled damage.

The failure distributions in Figure 7 were plotted on Normal (Gaussian) probability paper. The question whether Gaussian distributions provide the best fit led to a comparison of Gaussian with the Weibull and Kies distribution functions using the "best straight line fit" as a criterion. Table III shows the occurrence of the best fit among the three distribution functions. Where the fit was equally as good for more than one distribution, each was scored in the Table. As far as the entire test length range is concerned, the Gaussian distribution had approximately the same number of successes as the Kies, but more of these successes were in the good and good-to-fair fit classes. More significant is the fact that the majority of the best fits for the Gaussian distribution occurred at lengths above 1.5 cm, particularly in large sample sizes. At these test lengths, single flaw populations were mainly responsible for failure as evidenced by the constant log strength-log length slopes. At lengths shorter than 1.5 cm, mixed flaw populations were to be expected in consequence of the change in the strength-length slopes, and the Gaussian distributions should, therefore, be non-linear in this region. This is the case as demonstrated in Figure 7. Because of this result, it appeared that the Gaussian distribution provided the best method for analysing data to determine bi-modal distributions in the transition zone. However, the strengths at 50 percent probability of failure were nearly the same for all distributions. The choice of Gaussian, Kies, or Weibull distribution did therefore not affect the average strength value used in the strength-length plots.

Further analysis of distribution curves suggested a relationship between coefficient of variation, average fiber strength, test length, and surface conditions of the fibers. Data concerning 994 fibers were reviewed in this respect, and coefficients of variation were plotted in the strength-length graph (Fig. 9), at the location of their respective strength values. The accompanying legend shows the coefficient ranges selected, while the contour lines represent average values. Virgin fiber data and the 0.025 cm Group A test point from S-994 (9/19/62) lie on the 5 percent contour

TABLE III

OCCURRENCE OF GAUSSIAN, WEIBULL AND KIES DISTRIBUTIONS RELATED TO FIBER STRENGTH, BASED ON BEST STRAIGHT LINE FIT

TEST LENGTH, (cm)

Fiber	.025	.05	.1	.25	.5	.75	1	1.5	3	6	12	24	30	Average Degree of Fit
E-Glass								G	G	G W K	G	G	G	Good
X-994 S-I						W K		G	W K	G W	G	G K K		Good to Fair
X-994 S-II		G	K	W		G W		G	G					Good to Fair
X-994 Monofil.				none	none			K			K			Good to Fair
S-994 (July 1962)	G	K	G	G	W K	G W K		G G	K	W K				Fair to Poor
S-994 (9/19/62)	W K	W K	W K	W K	W K	K	G W	K	K	K				Fair
S-994 Undulated Straight					K G K									Fair to Poor

	.025	.05	.1	.25	.5	.75	1	1.5	3	6	12	24	30		Total	Total (1.5-30 cm)
Gaussian	1	1	1	1	1	2	1	5	2	2	2	2	1		22	14
Weibull	1	1	1	2	2	3	1		1	3					15	4
Kies	1	2	2	1	4	3		2	3	3	1	2			24	11

SAMPLE SIZES

Fiber	.025	.05	.1	.25	.5	.75	1	1.5	3	6	12	24	30
E-Glass								79	50	79	83	79	25
X-994 S-I						50		50	50	50	43	25 50	
S-994 S-II		12	25	25	25	25		25	25				
X-994 Monofil.				25	25			25			20		
S-994 (July 1962)	21	23	25	25	25	25		25 25	25	25			
S-994 (9/19/62)	23	25	25	25	25	25	25	25	25	25			
S-994 Undulated (9/19/62) Straight					50 50								

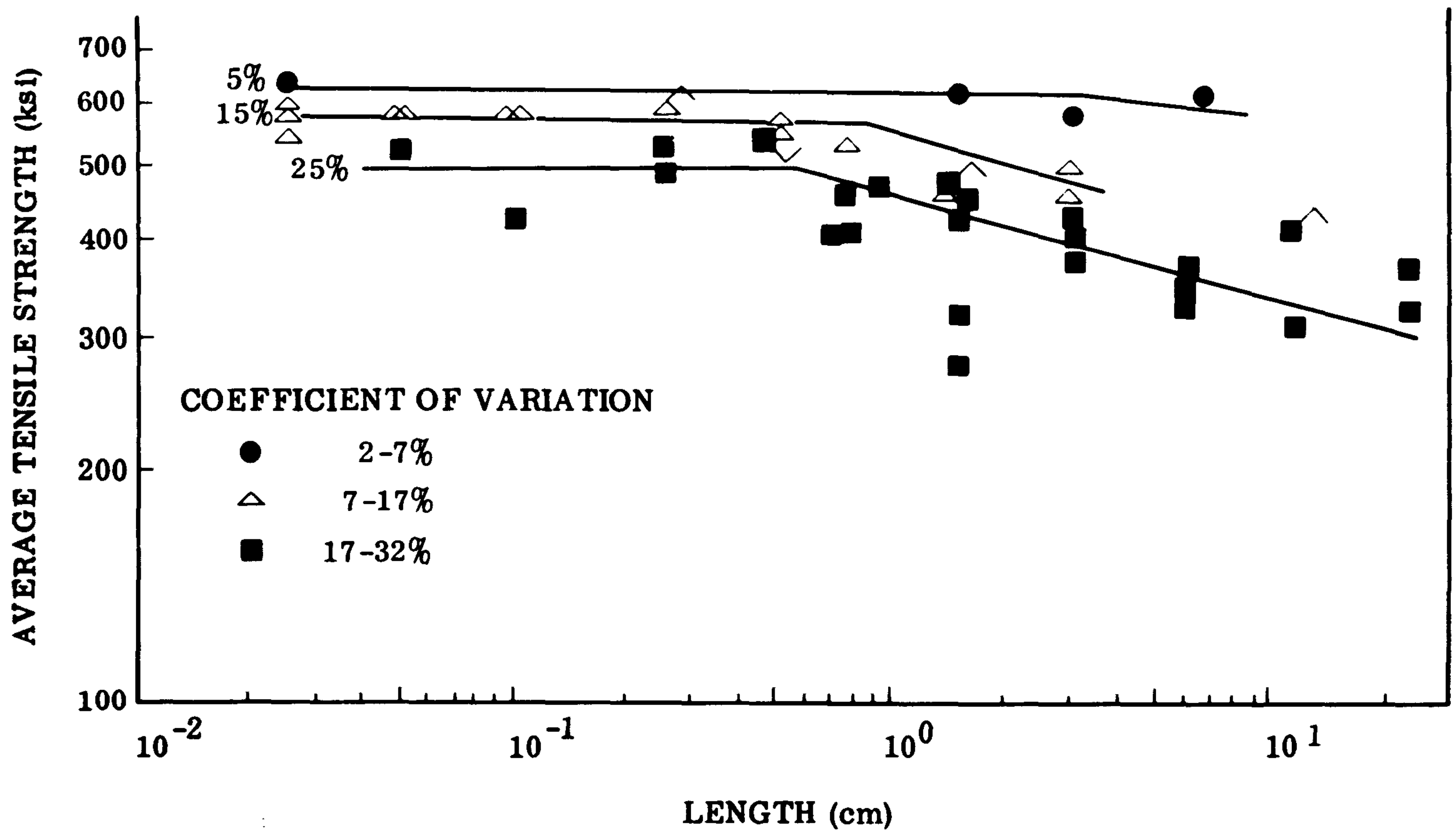

FIGURE 9. COEFFICIENTS OF VARIATION AS A FUNCTION OF FIBER DAMAGE AND LENGTH

line; all others are strand fiber data except for four data points from uncoated monofilaments from cardboard drum (flag on symbols). It can be seen that the coefficient depends directly on the degree of damage and only indirectly on length. This dependence may lead to relationships suitable for failure prediction at various confidence levels. More experimental work is necessary to substantiate the trends established.

In summary, bi-modal failure distributions are associated with a distinct slope change in the Gaussian failure distribution plots. Uniform Gaussian distributions occur relatively frequently at fiber lengths where single flaw populations are expected. Average fiber strength values, which are used in the strength-length plots, are somewhat independent of distribution functions. A tentative correlation of coefficients of variation with fiber strength data suggests another method of data analysis.

4.3 UPPER AND LOWER LIMITING STRENGTH

The upper limiting strength, σ_o, and lower limiting strength, σ_u, have been used by Weibull and Kies to describe probability of failure functions. The upper limiting strength was derived empirically from slope extrapolations (Ref. 7) of strength-length plots and was found to be twice the highest average strength. This led to the expression $\bar{\sigma} = \sigma_o/2$ limit $L \longrightarrow 0$. In view of the importance of the two limits for the theoretical treatment of failure distributions and for the Kies strength parameter, $\bar{\sigma} / \sigma_o - \bar{\sigma}$, test results were analyzed with respect to these two limits.

4.3.1 Upper Limiting Strength, σ_o

There are several approaches to the analysis of the data to determine the existence of an upper limiting strength. One of the simplest approaches is to consider the condition at the change in slope of the strength-length plot (Fig. 4). For the short-fiber behavior,

$$S_1 = 1 - \exp\left(-L_1 \left(\frac{\sigma_1}{\sigma_o}\right)^{m_1}\right)$$

and for the long-fiber behavior,

$$S_2 = 1 - \exp\left(-L_2 \left(\frac{\sigma_2}{\sigma_o}\right)^{m_2}\right)$$

where m_1 and m_2 are the slopes of the respective portions of the curve. But the strengths are determined at a probability close to 50 percent so that $S_1 = S_2$; hence,

$$L_1\left(\frac{\sigma_1}{\sigma_o}\right)^{m_1} = L_2\left(\frac{\sigma_2}{\sigma_o}\right)^{m_2}$$

At the change in slope, $L_1 = L_2$ and $\sigma_1 = \sigma_2$

This equation is meaningless if the upper limiting strength values for both short and long fibers are the same. To fit the experimental data, it is necessary to assume different values of the upper limiting strength for each portion of the curve. Consequently, it can be concluded that the upper limiting strength is essentially an adjustable constant to fit the data to the assumed distribution.

In an attempt to determine the maximum strength that can be expected with present glass fibers, the results of over 1000 tensile tests on 994 fibers were examined. Table IV summarizes the frequency of occurrence of strength levels above 700,000 psi. These point to a maximum strength of 800,000 psi for this fiber. Since 700,000 psi did not occur at test lengths greater than 0.5 cm (steep slope portion), the actual number of tests involved is 660.

TABLE IV

FREQUENCY OF OCCURRENCE OF HIGH STRENGTH VALUES

Strength (ksi)	Number of Occurrences
800 to 780	1
779 to 760	2
759 to 740	1
739 to 720	3
719 to 700	17

Further information on maximum strength values at the various test lengths is contained in Table XI of Appendix C.

4.3.2 Lower Limiting Strength, σ_u

An attempt was made to determine whether a lower strength limit other than zero is being approached as fiber length increases. For this purpose, slopes of the distribution curves (Gaussian function) were extrapolated to 0.01 percent probability of failure[1]. The respective strength values versus fiber length are plotted in Figure 10; scatter is due partly to small sample sizes. The expected trend towards lower strength with increasing fiber length is apparent; however, data from the different test series do not converge to a limiting value. The present assumption of zero strength as a lower limit therefore remains valid.

The lowest strengths actually measured were 105,000 psi for E-glass (at a probability of failure of 1 percent) and 94,000 psi (at a probability of failure of 2 percent) for 994 glass. A summary of minimum strength values for all tests on 994 glass is given in Table XI, of Appendix C and the frequency of occurrence of values below 140,000 psi is listed in Table V. These strength data occurred at fiber lengths between 3 and 30 cm.

TABLE V

FREQUENCY OF OCCURRENCE OF LOW STRENGTH VALUES

Strength (ksi)	Number of Occurrences 994	E
80 to 99	1	0
100 to 119	2	3
120 to 139	2	8

4.4 MECHANICAL FIBER DAMAGE

Figure 11 presents the pattern of behavior expected for glass fibers. This differs in minor aspects from the model presented by Kies (Fig. 1) because of the finding that the fracture behavior is not controlled by a single population of flaws.

[1] In case of bi-modal distributions, the lower tail end was extrapolated.

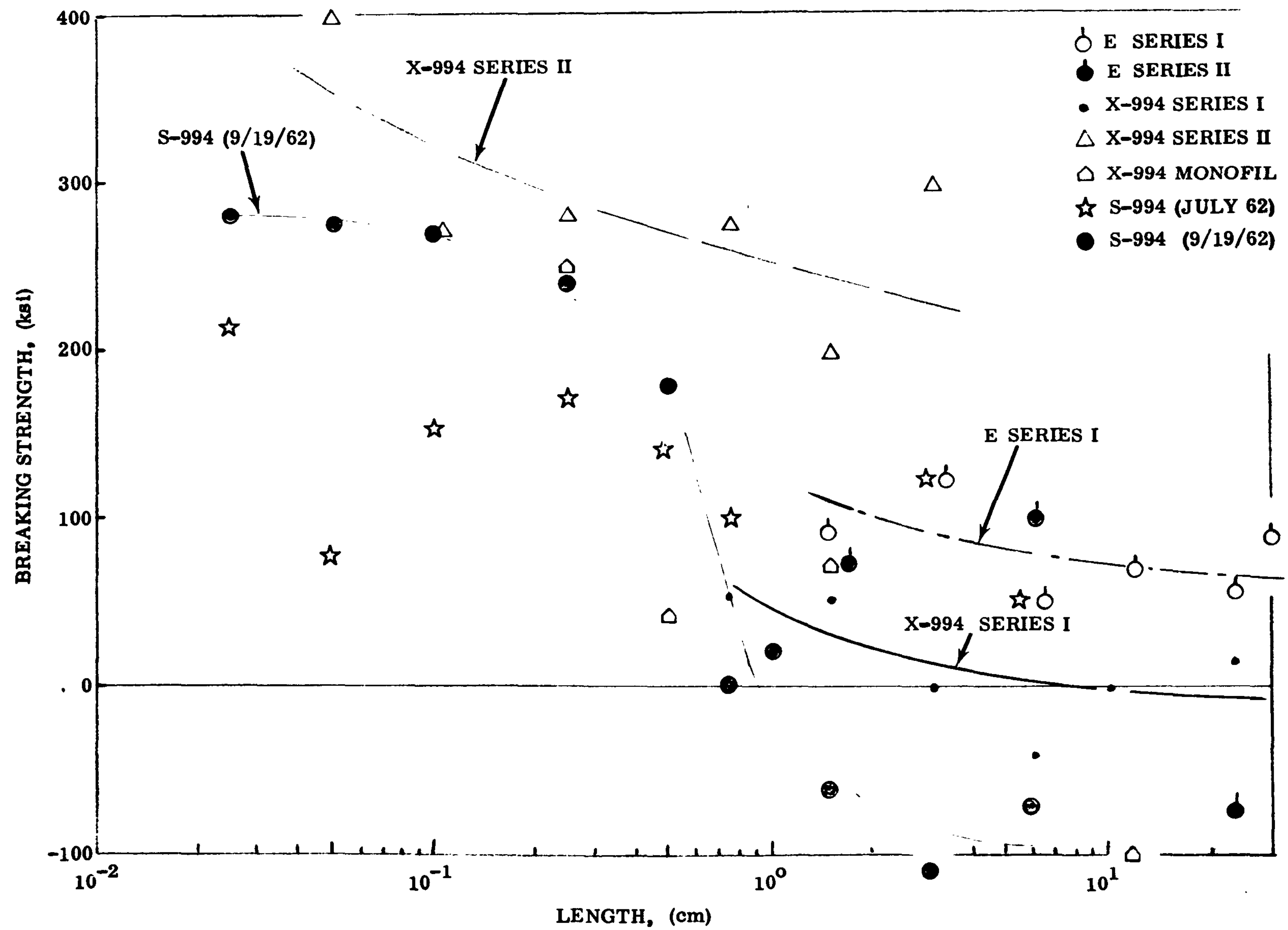

FIGURE 10. FIBER STRENGTH AT 0.01 PERCENT PROBABILITY OF FAILURE AS A FUNCTION OF TEST LENGTH

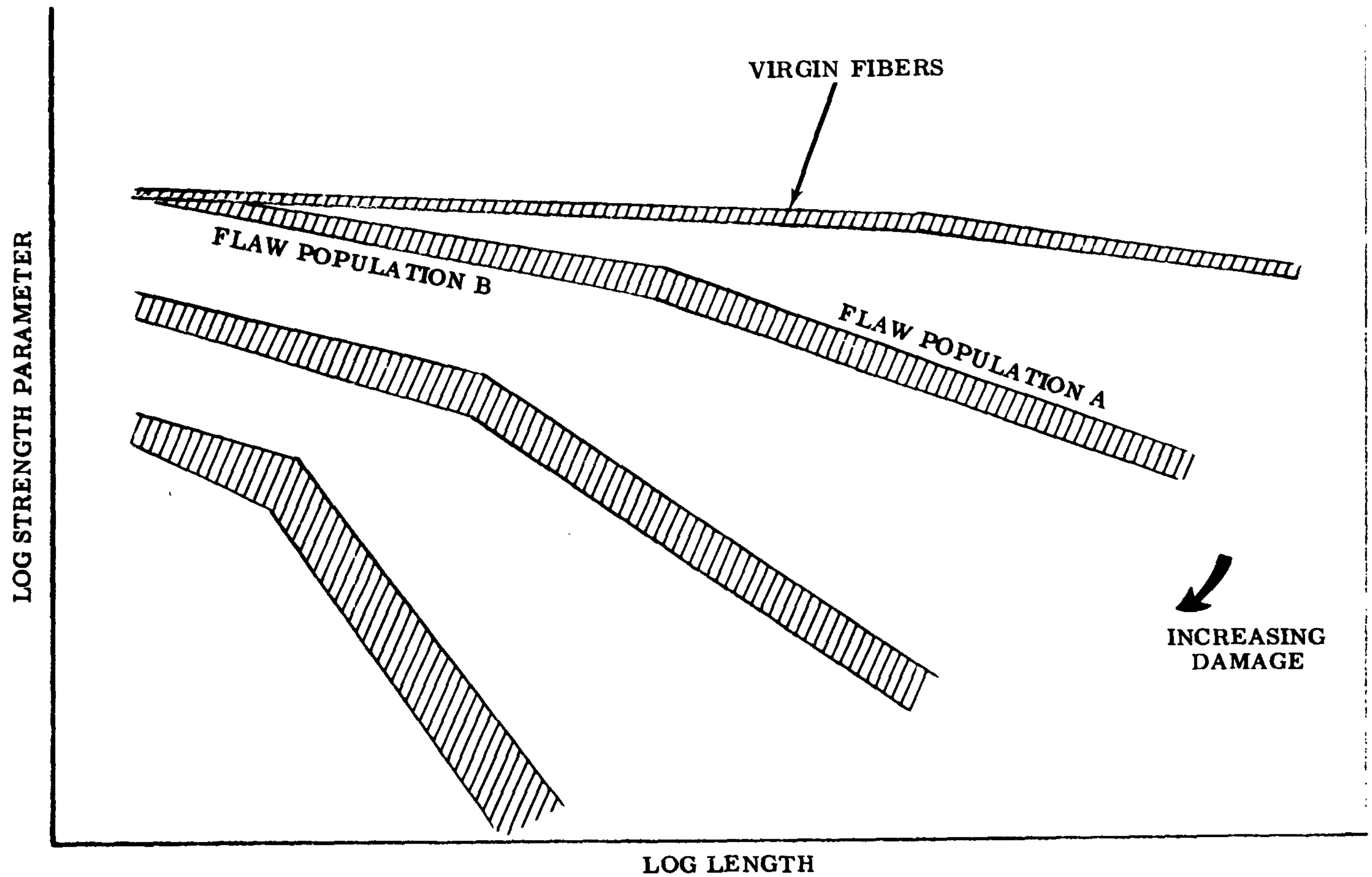

FIGURE 11. LENGTH EFFECT ON FIBER STRENGTH - MIXED SURFACE FLAW POPULATIONS (MODEL)

The effect of increasing damage is to increase the slopes of the logarithmic strength-length plot in both the short and long fiber lengths. In addition, the position of the change in slope moves toward shorter fiber lengths as the damage increases.

One observation deserves notice. When the finish is abundant at the edges of a strand, then fibers separated from the edge have strength values similar to or higher than those from the center; but when the amount of finish is marginal, edge fibers tend to have lower strength. These results suggest that the amount of mechanical damage in handling depends on the amount of finish available to protect the fiber. If sufficient finish is available for protection, the exposed edge fibers may have higher strengths than the less exposed center fibers where finish is inadequate for protection. This points to the need for better control of the distribution of finish to protect the strand uniformly.

The Kies and Weibull damage coefficients have been determined for the two populations of flaws (Fig. 11). Table VI gives these damage coefficients. Very high values of the damage coefficient (indicating negligible damage) were observed with virgin fibers. For example, extremely careful work by Thomas (Ref. 8) shows almost no variation with area of fibers and hence the damage coefficient approaches infinity[1]. On the other hand, more typical values for virgin fibers show m and α values approximately equal to 15 and 10 respectively.

For long fibers separated from strands, there is a remarkable constancy of values for both E-glass and 994 glasses. This consistency might be expected from the following analysis. At a length ℓ_1, the flaw at 50 percent probability of failure will have a stress concentration factor of k_1. If the theoretical strength of the glass is σ_{max}, then failure will occur at σ_{max}/k_1. Using the Weibull approach for example, the slope m is given by,

$$m = -\frac{\Delta(\log \ell)}{\Delta(\log m)}$$

Substituting σ by σ_{max}/k gives:

$$m = -\frac{\log \ell_1 - \log \ell_2}{\log k_2 - \log k_1}$$

[1] Thomas's data appear as a single point in Figure 3 because he used constant test length and varied the diameter over a wide range from 25 to 60 x 10^{-5} inch.

TABLE VI

DAMAGE COEFFICIENTS OF E-GLASS AND 994 GLASS

Flaw Population	Symbol (Fig. 3 and 4)	Glass		Weibull m	Kies a	Remarks
B	△	E (Otto)	Virgin	15	12	
	Not Applicable	E (NRL Report 5098)	Virgin, paraffin coated	11.2	10	Adjusted to $_o$ = 1,100 ksi
	△	X-994	Cardboard drum	19	11	Low foil end
	●○	E (Series I and II)	From strands	6.2	4.8	
	•△	X-994 (Series I and II)	From strands	6.4	5	
	●	S-994 (9/19/62)	From strands	6.4	5	
A	◊	X-994	U-Frames	~150	~100	Slope less well defined due to data scatter.
	△	X-994 (Series II)	From strands	~100	~ 60	
	●	S-994 (9/19/62)	From strands	27	16.5	

<u>NOTES</u>:

Length unit of m and a is centimeter.

All slopes are negative.

Damage increases as numerical values decrease.

E-glass, δ_o = 1,100,000 psi.

994 glass, δ_o = 1,400,000 psi.

Hence, the slope is independent of the theoretical strength of the glass, but is related to the variation in stress concentration factors of flaws with length. Accordingly, for the same damage, the same value of m is expected for both E and 994 glasses. The constancy of values reflects the constancy of damaging factors in the manufacture and handling of strands.

4.5 RELATIVE PROPERTIES OF GLASSES

Several factors influence the assessment of the relative merits of different glasses. The foremost factor is the strength, but density, elastic modulus, and the tendency to become damaged are also important.

Considering the strength first, data of the two glasses are listed in Table VII for virgin fibers and fibers separated from strands. Each column contains values for three different test lengths. The strength ratio, σ_{994}/σ_E, is the quantity desired for strength comparison.

TABLE VII

COMPARISON OF STRENGTH OF 994 AND E-GLASS

	Virgin, $m > 15$	From Strand, $m = 6.2$ and 6.4		
Length (cm) →	2.5	1.5	6	24
Strength of:				
E	530	330[1]	265	215
994	610	450[1]	365	300
Ratio σ_{994}/σ_E	1.14	1.36	1.38	1.39

[1] Strength values from mean slopes of different test series.

On the basis of strength, the 994 strand fibers are superior by a factor of approximately 1.4. The strength ratio of virgin fibers is less reliable because very few virgin 994 fibers were available for test.

Since strength is but one of the fiber properties, inclusion of elastic modulus, E, and density, ρ, as descriptive factors was proposed by Kies, using the relationship:

$$\text{property factor, } p = \frac{E}{\rho}\,\sigma$$

E/ρ is a constant for a given glass and its value for both E and 994 glasses is:

	E	994	Units
Elastic Modulus	10.5×10^6	12.2×16^6	psi
Density	0.092	0.0875	lb/inch^3
E/ρ	1.14×10^8	1.39×10^8	inch

The ratio $(E/\rho)994/(E/\rho)E$ of the two glasses is 1.21.

Multiplication of the strength ratios, $\sigma\ 994/\ \sigma_E$, in Table VII by 1.21 gives the desired property ratios, $p_{994}/\ p_E$, listed in Table VIII.

TABLE VIII

COMPARISON OF PROPERTIES OF 994 AND E-GLASS

	Virgin, m > 15	From Strand, m = 6.2 and 6.4		
Length (cm)	2.5	1.5	6	24
$p_{994}/\ p_E$	1.39	1.64	1.67	1.69

The 994 fiber properties are more favorable on this basis than on strength alone. It is interesting to compare the property ratio of strand fibers with the performance increase of Polaris cases after introduction of 994 glass; this increase, based on burst strength, was 1.5.

The effect of damage on the relative properties of the two glasses is difficult to assess because of the lack of short fiber E-glass data on strands, and lack of informative 994 data from freshly drawn fibers. The property factor ratio, p_{994}/p_E, of the strand fibers is nearly constant over the length range available for comparison. This consistency is to be expected because the degree of damage, expressed by the slope, m, is nearly the same for the two strands. This similarity is discussed in Section 4.4. Consequently, if typical damage and resultant strength loss differ for the two fibers, the property ratio would change with length; at some length, the advantage of one fiber over the other would disappear. In terms of strength, this point would be reached at a strength ratio $\sigma_{994}/\sigma_E = 0.82$ which corresponds with the property ratio, p_{994}/p_E, equal to unity.

In conclusion, the 994 fibers are superior to E-glass fibers by a factor approaching 1.4 if strength is considered, and nearly 1.7 if strength, elastic modulus, and density are taken into account. Both values refer to strand fibers at gage lengths larger than 1.5 cm. The respective factors appear to be lower for virgin fibers on the basis of limited evidence; however, it is probable that values equal to those from strand fibers may be reached. The advantage of 994 over E-glass would disappear at a strength ratio of 0.82 when the property ratio becomes unity. This might occur if unfavorable slope changes result from change in damage characteristics of either fiber; for instance, in the short fiber range which has not yet been explored for E-glass.

4.6 EFFECT OF SAMPLE SIZE ON AVERAGE STRENGTH

Most of the average fiber strength data were obtained from sample sizes of approximately 25 fibers, which is generally considered to be sufficient for reliable average strength values. Since sample sizes of 50 were used in some tests, a convenient means of checking the reliability of smaller groups was available. These larger samples were divided into two groups: Group A - fibers 1 through 25 (in order of separation), and Group B - fibers 26 through 50. Average strength was calculated for each group and then compared with the average of the total. Table IX shows the results of this investigation. The differences are small with the unexplained exception of the 6-cm data. Furthermore, there is no preference of one group over the other. Fiber separation, described in Section 2.4, did not introduce progressive damage as separation proceeded.

TABLE IX

EFFECT OF SAMPLE LIFE ON AVERAGE STRENGTH

Fiber	Test Length (cm)	Group	$\bar{\delta}$ (ksi)	Difference (%)	Remarks
X-994, Series I	0.75	A	462	± 1.7	
		B	447		
		A + B	455		
	1.5	A	450	± 3.7	
		B	419		
		A + B	435		
	3	A	380	∓ 1.2	
		B	396		
		A + B	388		
	6	A	382	± 10.2	
		B	310		
		A + B	346		
	12	A	323	± 5.8	
		B	292		
		A + B	310		
	24	A	324	∓ 0.5	
		B	327		
		A + B	325		
S-994, (9/19/62)	0.5	A	542	± 3.0	Undulated
		B	511		
		A + B	526		
	0.5	A	485	∓ 1.4	Straight
		B	499		
		A + B	491		

Note: Group A = Fibers 1 - 25 (in order of separation)
B = Fibers 26 - 50
A + B = Fibers 1 - 50

4.7 LENGTH EFFECT ON STRAND STRENGTH

To provide a link between the strength of single fibers and strands, an investigation of the length effect was extended to strands of the same batch from which fibers were separated for single fiber tests. A limited number of tests was conducted. Results are therefore considered to be preliminary, although some definite trends were established.

Data obtained from resin-free and resin-impregnated strands are listed in Table III of Appendix B, and are plotted in Figure 12.

The strength reversal at shorter gage lengths in the 994 plot led to a close examination of the load-elongation curves and it was found that poor fiber collimation was the cause. Relations could be established between progressive loading of poorly collimated fibers and elongation of the entire strand (Fig. 13). This has been discussed in Report 4 of Reference 5. It is obvious that different results will be obtained with strands having different fiber collimation and/or with change of pre-tension as was the case with the Owens-Corning strand tests. The observed length effect on strand strength should contribute to failure analysis of filament-wound pressure vessels, particularly where decoupling or separation of short strand length is likely to occur in the initial phase of rupture.

Divergent opinions exist over the question whether the average strength values from single-fiber tests can be compared with those from strand tests on the basis of equal gage length. In essence, the argument is whether the total glass length of the strand (i.e., of the 204 fibers) or the nominal gage length must be used in strength-length plots. Examination of the strand strength-length plots (Fig. 12) shows that resin-free strand strength is equal to the average (single) fiber strength in the case of X-994 glass at test lengths where the collimation effect is small. If the total fiber length of the 204 fiber strand were to be used, the measured strengths would have to be plotted more than two decades to the right, at approximately 2000 cm. It can be concluded that the basis of comparison for 994 glass must be gage length. On the other hand, the lower strand strength of Solar's tests on E-glass was due to a severe fiber collimation. Owens-Corning's data for 20-end rovings are plotted in Figure 12 and fall close to the single fiber curve determined at Solar. If total glass fiber length were the basis of comparison, the Owens-Corning's data would have to be plotted four decades to the right (400 meters total length).

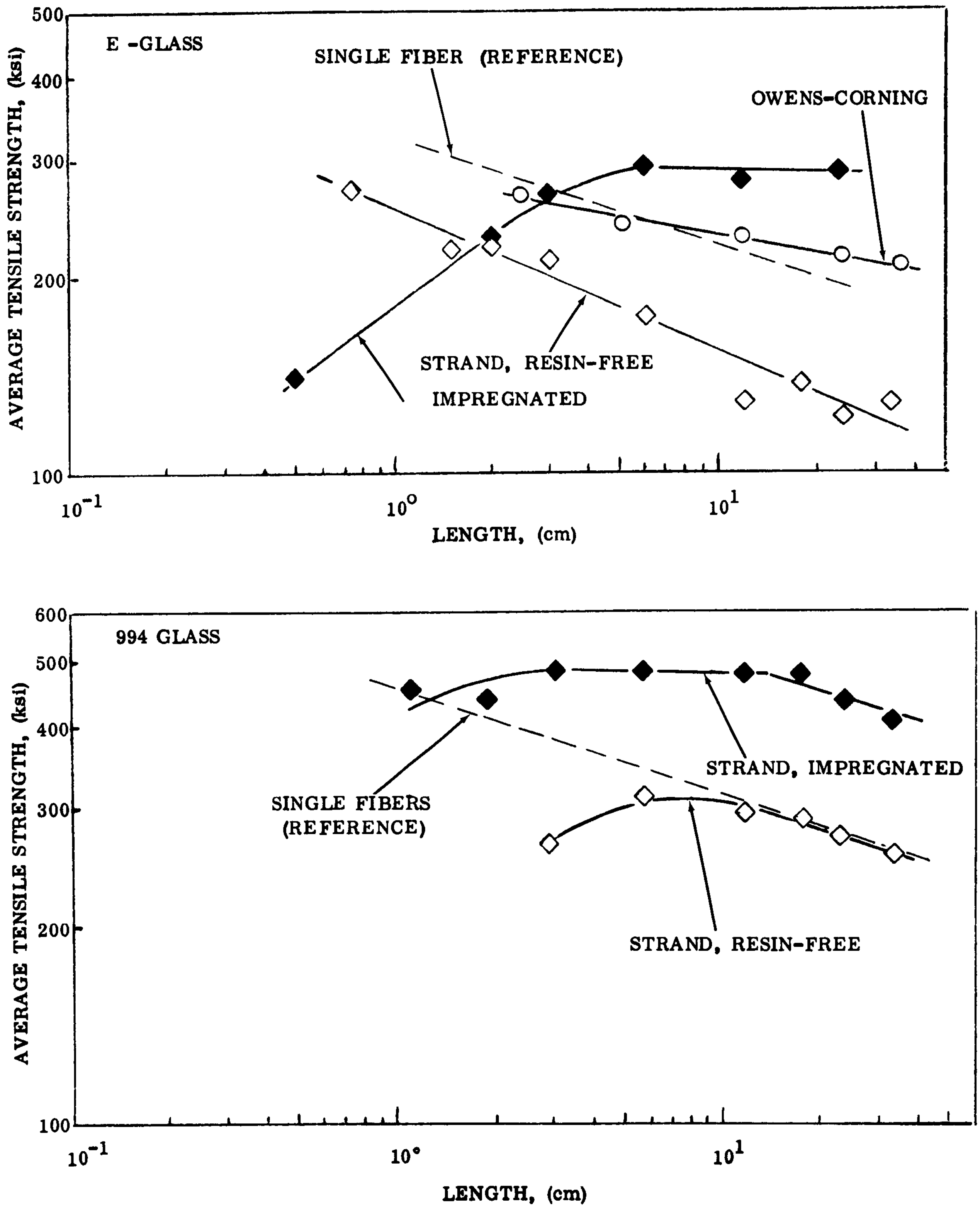

FIGURE 12. LENGTH EFFECT ON BREAKING STRENGTH OF STRANDS WITH AND WITHOUT RESIN

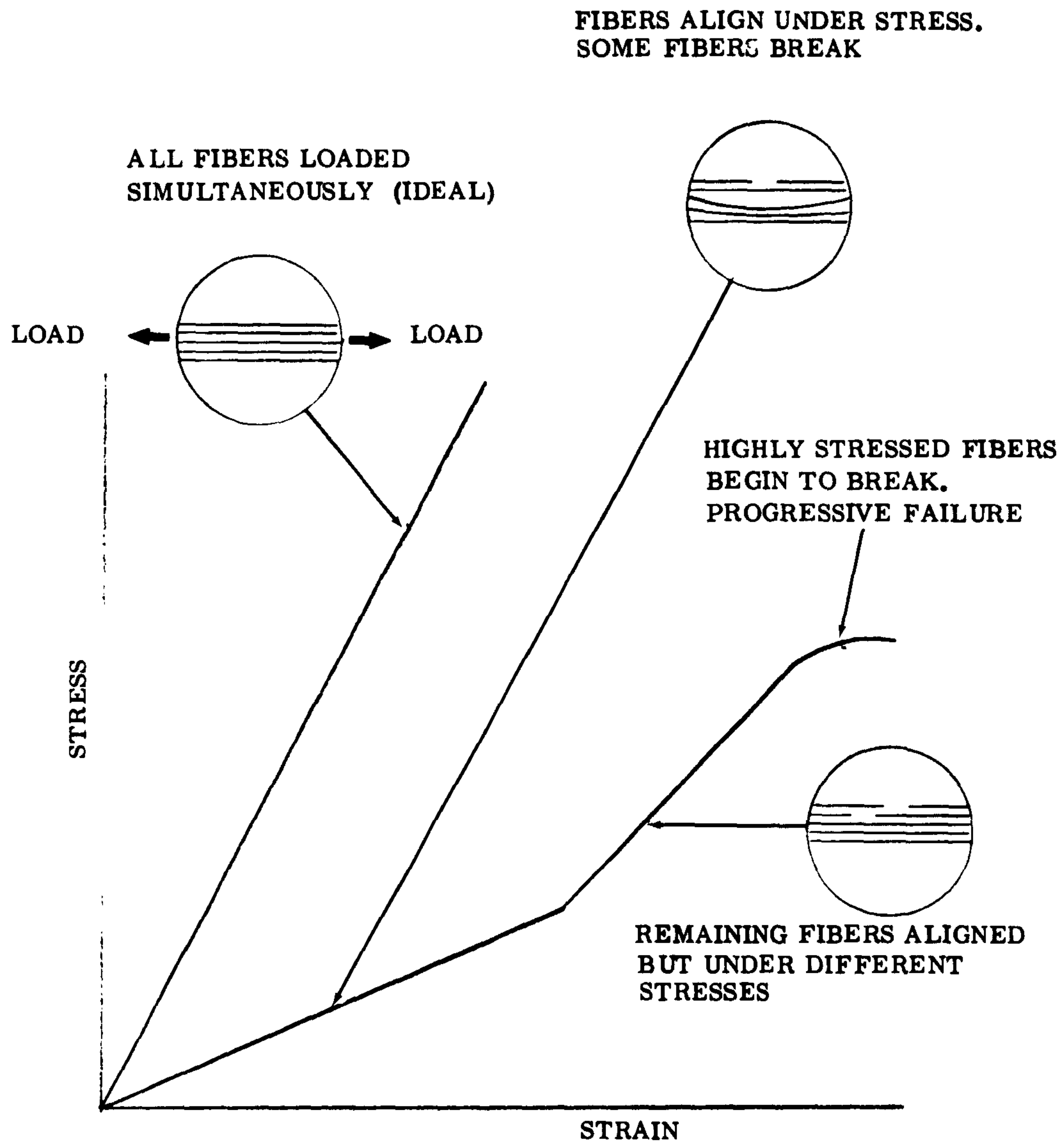

FIGURE 13. DEGRADING EFFECT OF POOR FIBER COLLIMATION ON STRAND STRENGTH

In conclusion, it has been shown that length effects exist for strands as well as for fibers. At short-strand lengths, a new effect appears related to the degree of collimation. At long-strand lengths, the effect is similar to that on fibers so that strengths match when each is tested at the same gage length.

V. CONCLUSIONS

1. The effect of length on fiber strength was demonstrated in tensile tests of monofilaments, strands, and individual fibers from strands.

2. Experimental data showed that more than one type of defect controls the strength of glass fibers.

3. Two distributions of defects were apparent from failure distribution plots, and caused a change in the strength-length slope.

4. An analytical method was devised to assess the bi-modal failure distributions and to relate these distributions to the change in the strength-length slope.

5. A physical model has been proposed to incorporate these findings.

6. A method was developed to determine the relative strength of different glasses on the basis of strength, weight, Young's modulus, degree of fiber damage, and length of fibers.

VI. RECOMMENDATIONS

The strength-length relationships presented are insufficient to extract a pattern of general validity. The primary reason is that no control has been possible on the degree of damage of fibers received from commercial vendors or directly from the manufacturer. Furthermore, it was not known whether the virgin fiber strength of fibers from the same glass had been the same.

A series of experiments are recommended in order to fully describe the strength properties of E and 994 glass fibers. Three degrees of damage should be investigated: as drawn, and two degrees of mechanical damage that must be controllable and reproducible. It is essential that the average virgin strength of all fibers be approximately the same in the freshly drawn condition. This condition presents a formidable task since large sample sizes are required for each length tested to secure sufficiently accurate failure distributions. The drawing of E-glass fibers at Solar during the contract period was in preparation for such a program. Drawing of 994 fibers has been accomplished and will be necessary unless a sufficient amount of suitable 994 fibers can be obtained from Owens-Corning.

It is further recommended that the experimental work be supplemented by:

- statistical analysis of fiber strength data
- development of mathematical relationships between flaw density and length
- surface studies to identify surface flaws using techniques such as decoration
- structural studies to identify structural defects by means of low-angle X-ray diffraction

Information thus obtained should lead to a reliable strength-length model. It is recommended that fibers from different sources be checked against this model by a number of selected tests. Comparative results are then an indication for fiber quality.

The demonstration of the effects of length on fiber strength remains of limited value unless related to the end product, the composite structure. It is therefore recommended that the length effect study be extended to strands and suitable composite test specimens; of particular interest are the effects of fiber collimation and resin impregnation on surface damage. A further recommendation is concerned with the problem of interfacial fiber-resin separation under actual load conditions. Knowledge of the de-bonded gage length then ties in with length effect studies on single fibers. Failure prediction will thus become more reliable.

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APPENDIX A

RESULTS FROM S-994 (9/16/62) FIBERS

Figure

Table

Fibers from an S-994 strand drawn on 9/19/62 were investigated. A three-foot long partial strand of approximately 45 fibers was separated and cut according to individual test lengths. This partial strand was taken from the center and not, as previously, from the edge of the strand for reasons given below. The length range was the same as in previous tests, 0.025 to 6 cm.

Results from the S-994 (9/19/62) tests are listed in Table A-1 and plotted in Figure A-1, log strength versus log length. The small amount of scatter allows a reliable strength-length curve to be drawn. The definite slope in the short gage length region suggests a flaw mechanism different from X-994.

An additional 0.025-cm point was obtained with different fibers from another location on this strand. This sample was comprised of 15 fibers from the edge (Group A) and 10 fibers more nearly from the center of the strand (Group B). Edge fibers were distinctly larger (39 to 45 x 10^{-5} inch) than the center fibers (35 x 10^{-5} inch). The edge fiber strength was 14 percent higher than the center fiber strength (Table A-1). It is believed that fiber strength was preserved by what appeared to be an excess amount of the coupling agent assuming approximately equal strength of the virgin fiber[1]. Fiber separation was more difficult. This effect in conjunction with the larger diameters, led originally to the choice of center fibers for the strength-length investigation of this strand.

Diameter distributions (Fig. A-2) are given for comparison with previous tests; namely, S-994 (July '62) in Report 5 of Reference 5, X-994 Series II in Figure A-3, and X-994 Monofilament in Figure A-4. Likewise, failure distributions plotted on normal probability paper are shown in Figure A-5.

Periodic undulations of part of this strand led to strength measurements on fibers from such a strand section. The most severe undulation, shown in Figure A-6, was selected and strength was compared with a straight strand section. The respective diameter distributions are given in Figure A-7, and failure distributions

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[1] In Solar Report 4 (Ref. 5), it was recorded that Owens-Corning had found a 3 percent strength variation across a strand.

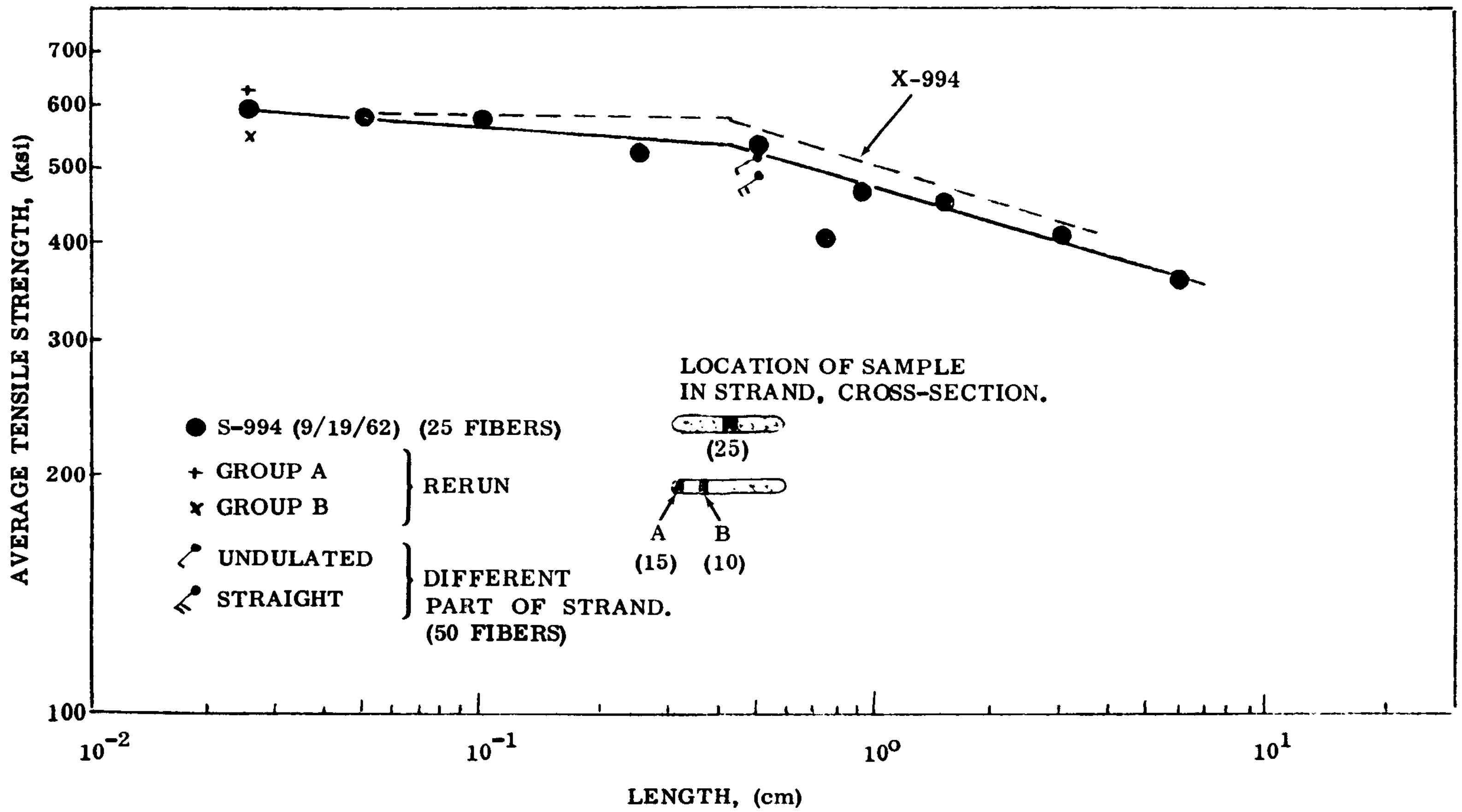

FIGURE A-1. LENGTH EFFECT ON BREAKING STRENGTH OF FIBERS FROM S-994 (9/19/62) STRAND

Fibers from an S-994 strand drawn on 9/19/62 were investigated. A three-foot long partial strand of approximately 45 fibers was separated and cut according to individual test lengths. This partial strand was taken from the center and not, as previously, from the edge of the strand for reasons given below. The length range was the same as in previous tests, 0.025 to 6 cm.

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An additional 0.025-cm point was obtained with different fibers from another location on this strand. This sample was comprised of 15 fibers from the edge (Group A) and 10 fibers more nearly from the center of the strand (Group B). Edge fibers were distinctly larger (39 to 45 x 10^{-5} inch) than the center fibers (35 x 10^{-5} inch). The edge fiber strength was 14 percent higher than the center fiber strength (Table A-1). It is believed that fiber strength was preserved by what appeared to be an excess amount of the coupling agent assuming approximately equal strength of the virgin fiber[1]. Fiber separation was more difficult. This effect in conjunction with the larger diameters, led originally to the choice of center fibers for the strength-length investigation of this strand.

Diameter distributions (Fig. A-2) are given for comparison with previous tests; namely, S-994 (July '62) in Report 5 of Reference 5, X-994 Series II in Figure A-3, and X-994 Monofilament in Figure A-4. Likewise, failure distributions plotted on normal probability paper are shown in Figure A-5.

Periodic undulations of part of this strand led to strength measurements on fibers from such a strand section. The most severe undulation, shown in Figure A-6, was selected and strength was compared with a straight strand section. The respective diameter distributions are given in Figure A-7, and failure distributions

[1] In Solar Report 4 (Ref. 5), it was recorded that Owens-Corning had found a 3 percent strength variation across a strand.

TABLE A-I

TEST DATA SUMMARY

S-994 (9/19/62), DRUM 1

	L (cm)	L (in.)	D (10^{-5} in.)	δ (ksi)	Standard Deviation (ksi)	Coefficient of Variation (%)	Number of Samples	Environments (F)	Environments (%RH)
	0.025	0.01	38.0	597	76.4	12.8	23	76	48
	0.05	0.02	38.5	579	70.0	12.1	25	78	40
	0.1	0.04	36.8	572	85.0	14.8	25	76	44
	0.25	0.1	38.0	526	92.8	17.7	25	76	52
	0.5	0.2	38.8	533	98.4	18.5	25	74	50
	0.75	0.3	38 2	404	101.6	25.1	25	74	50
	1.0	0.4	38.9	463	116.3	25.1	25	74	50
	1.5	0.59	37.1	452	134.7	29.8	25	76	48
	3	1.18	38.6	408	125.8	30.8	25	76	65
	6	2.36	36.9	364	90.3	29.8	25	76	48
Total	0.025[1]	0.01	41.3	598	71.3	11.5	25	74	45
Group A			43.8	633	36.9	5.8	15		
Group B			37.8	545	77.4	14.2	10		

[1] Different part of strand.

TABLE A-II

UNDULATED SECTION VERSUS STRAIGHT STRAND SECTION

	L (cm)	L (in.)	$\bar{D}$ (10^{-5} in.)	$\bar{\sigma}$ (ksi)	Standard Deviation (ksi)	Coefficient of Variation (%)	Number of Samples	Environments (F)	Environments (%RH)
Undulated	0.5	0.2	39.2	526	100.5	19.1	50	76	48
Straight	0.5	0.2	38.2	491	76.9	15.6	50	74	50

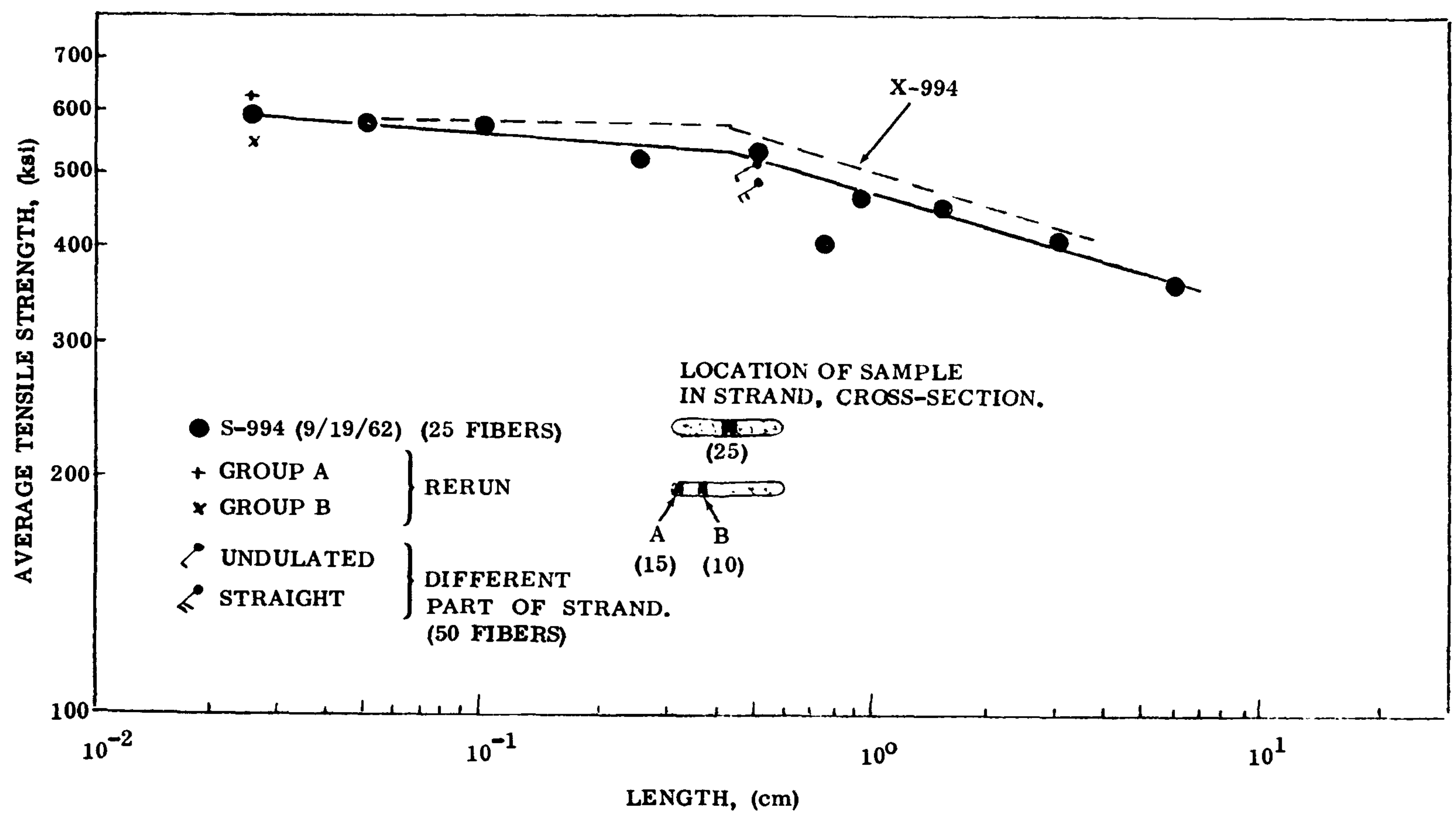

FIGURE A-1. LENGTH EFFECT ON BREAKING STRENGTH OF FIBERS FROM S-994 (9/19/62) STRAND

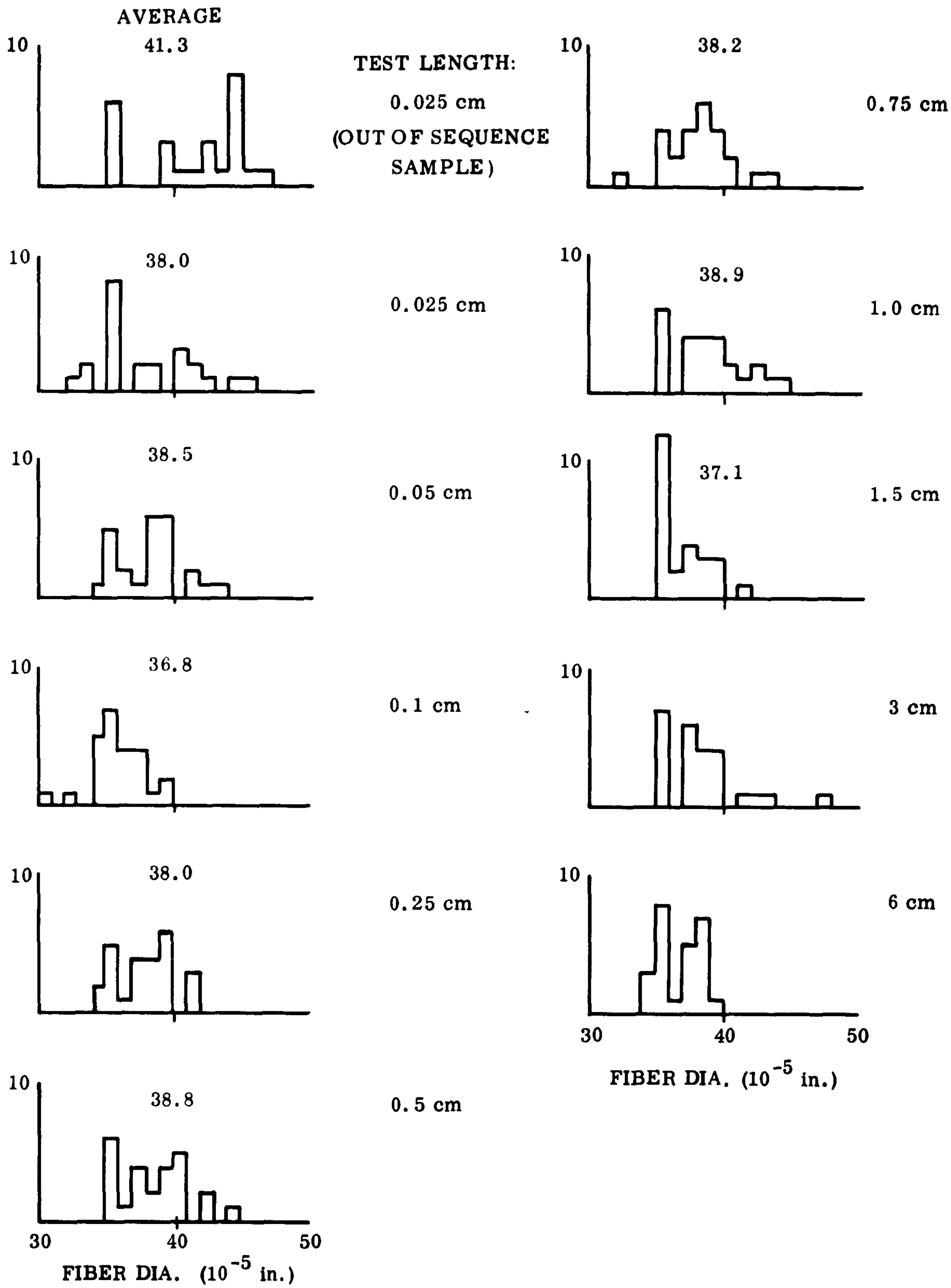

FIGURE A-2. DIAMETER DISTRIBUTIONS OF DIFFERENT TEST LENGTHS FROM PARTIAL STRAND S-994 (9/19/62)

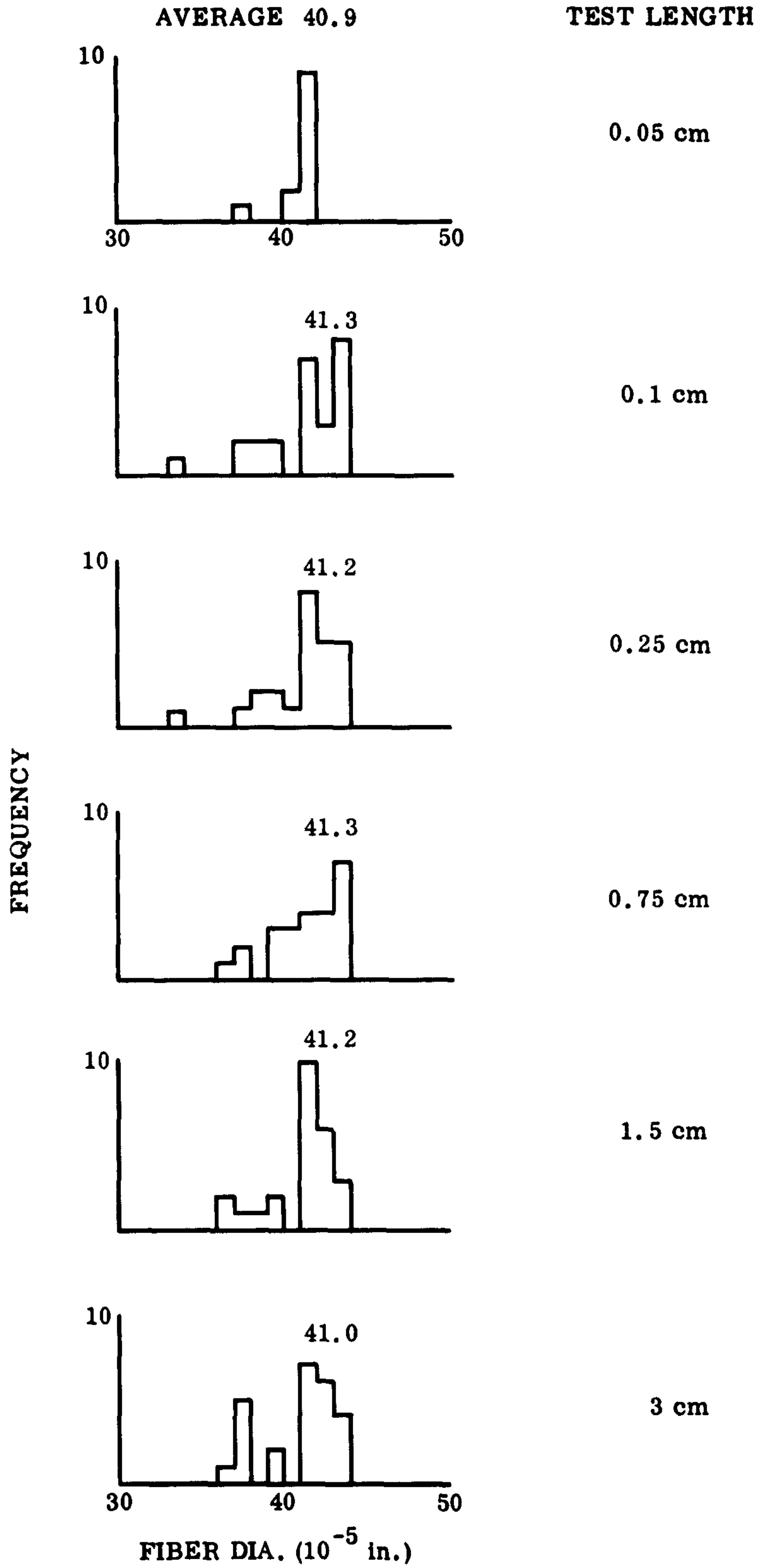

FIGURE A-3. DIAMETER DISTRIBUTIONS OF DIFFERENT TEST LENGTHS FROM X-994, SERIES II

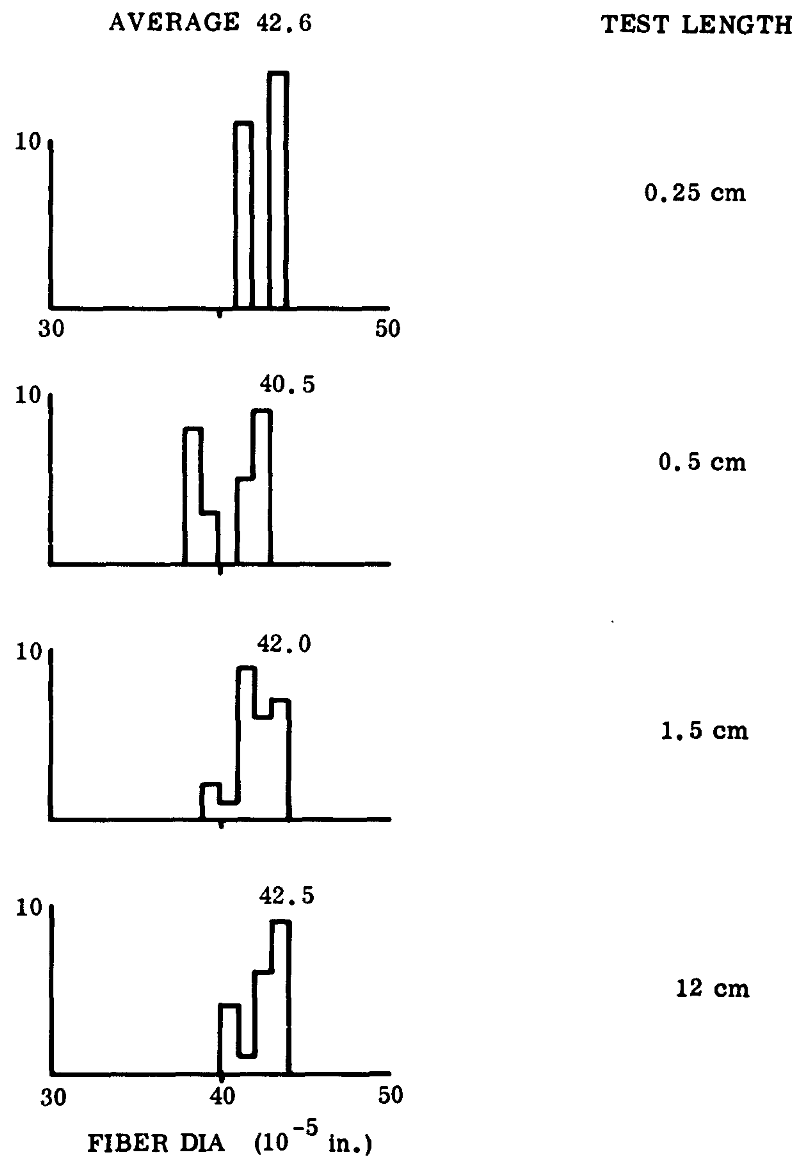

FIGURE A-4. DIAMETER DISTRIBUTIONS OF DIFFERENT TEST LENGTHS FROM X-994 MONOFILAMENT

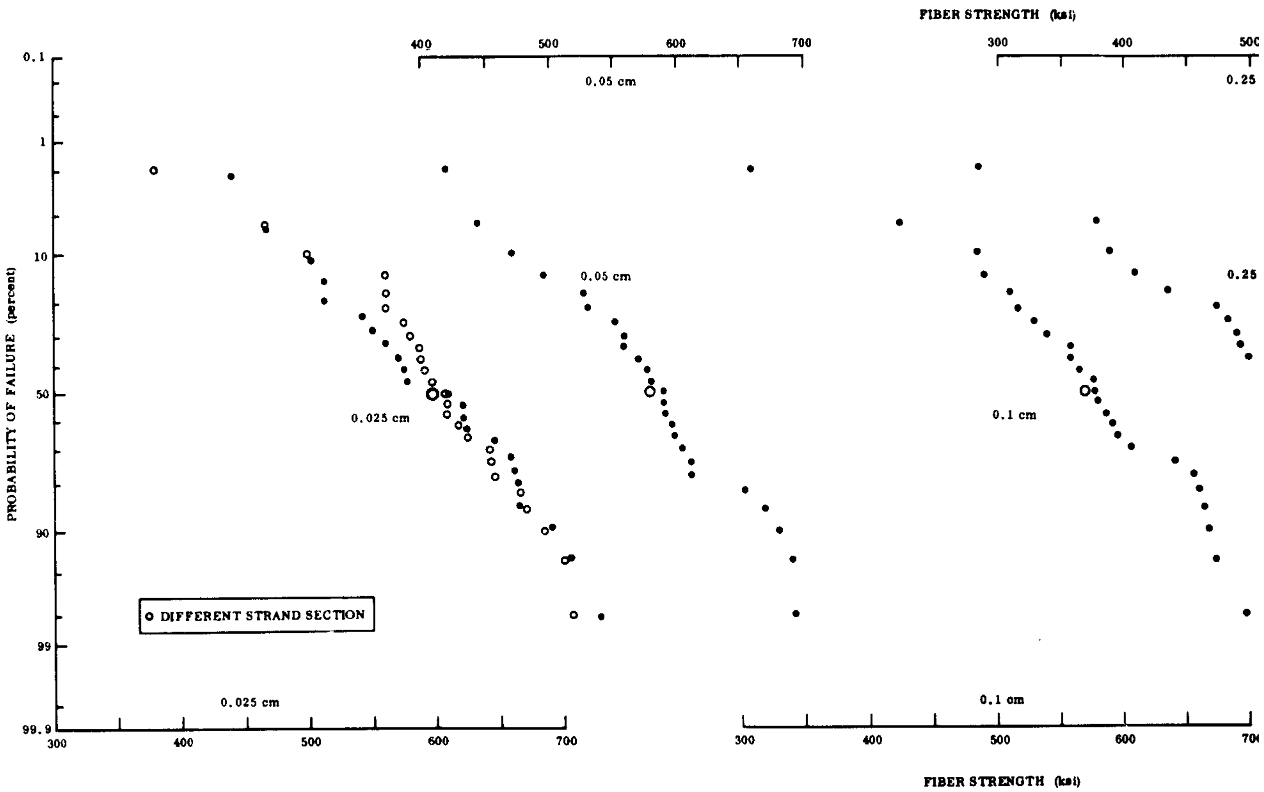

**FIGURE A-5. FAILU
FROM**

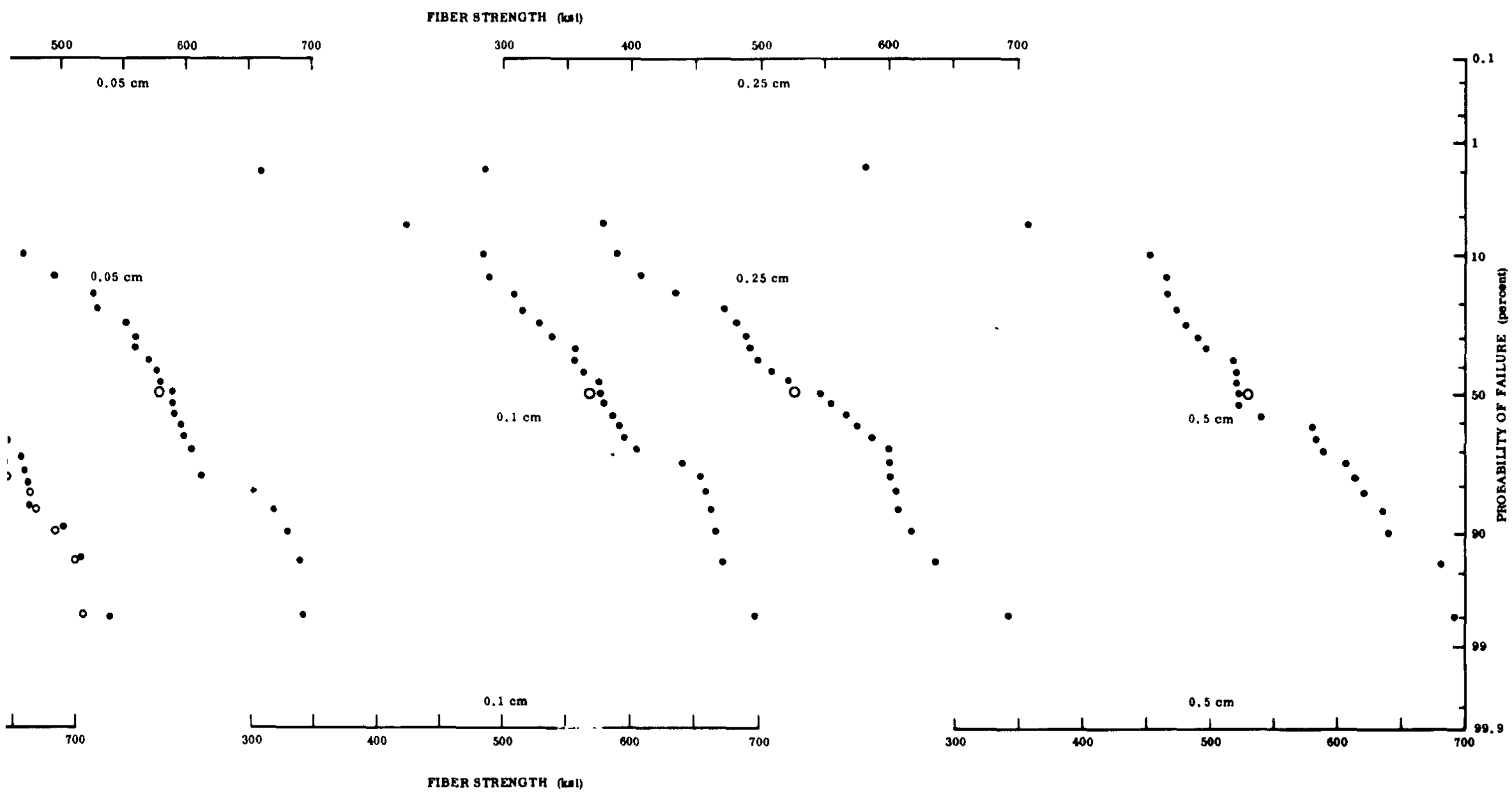

FIGURE A-5. FAILURE FREQUENCY DISTRIBUTIONS OF FIBERS SEPARATED FROM S-994 (9/19/62) STRAND

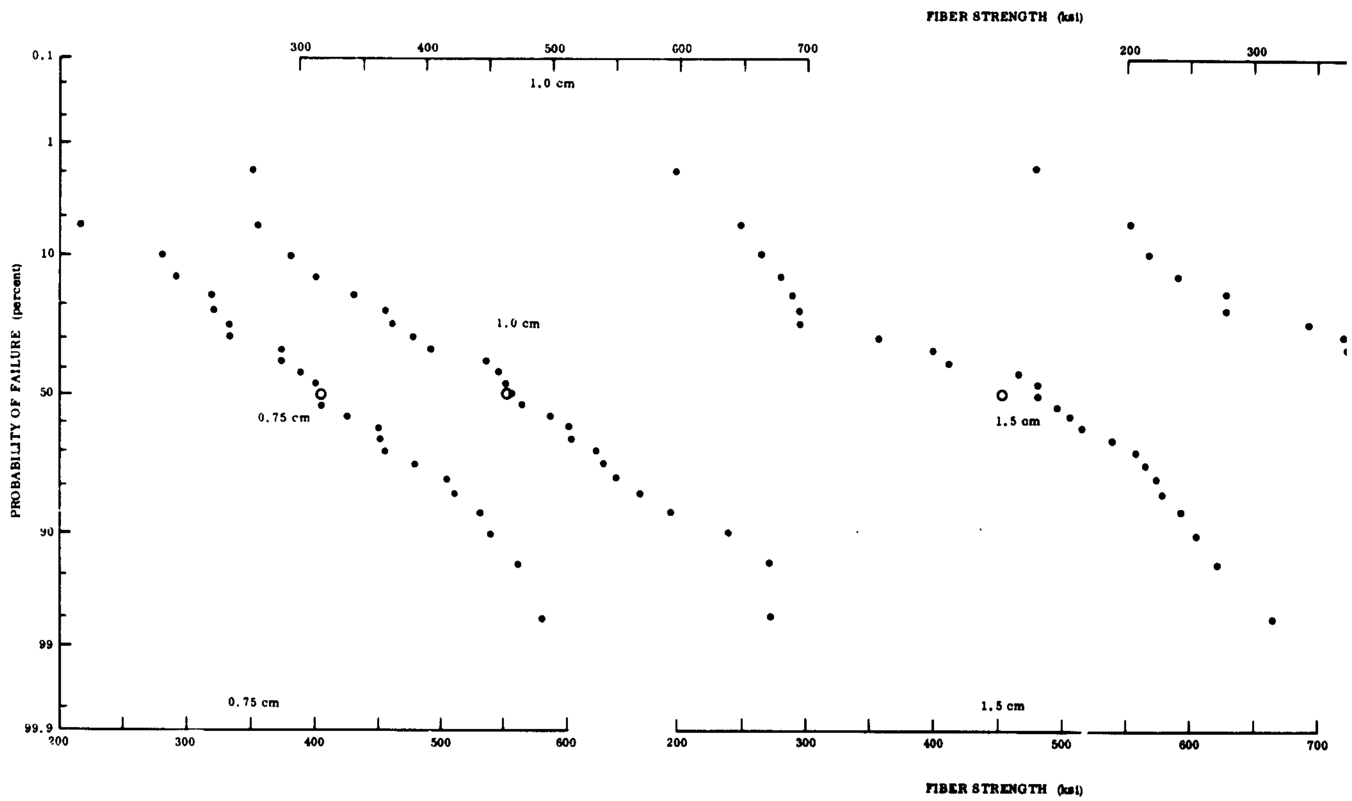

**FIGURE A-5. FAILURE FI
FROM S-994**

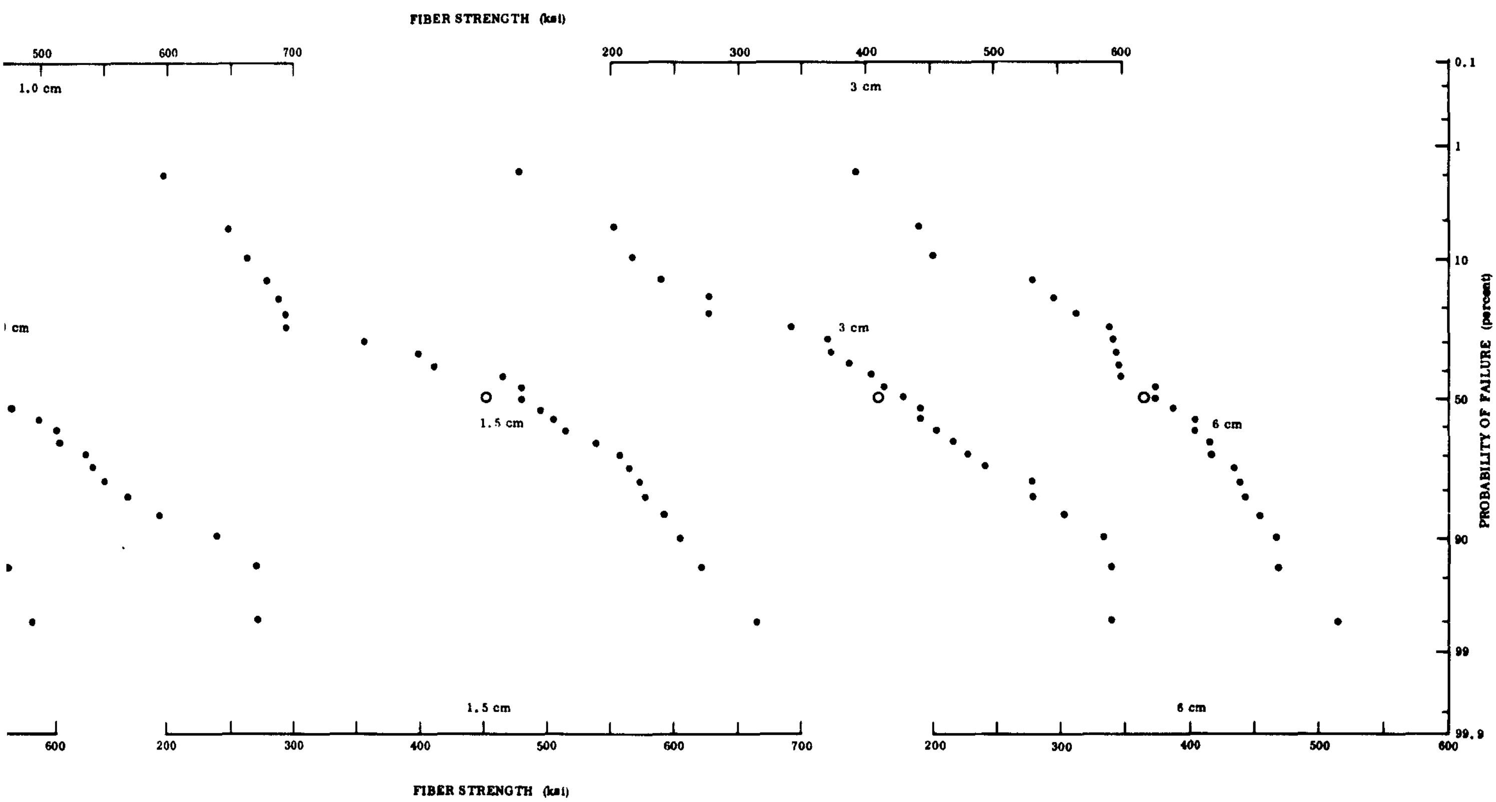

FIGURE A-5. FAILURE FREQUENCY DISTRIBUTIONS OF FIBERS SEPARATED FROM S-994 (9/19/62) STRAND

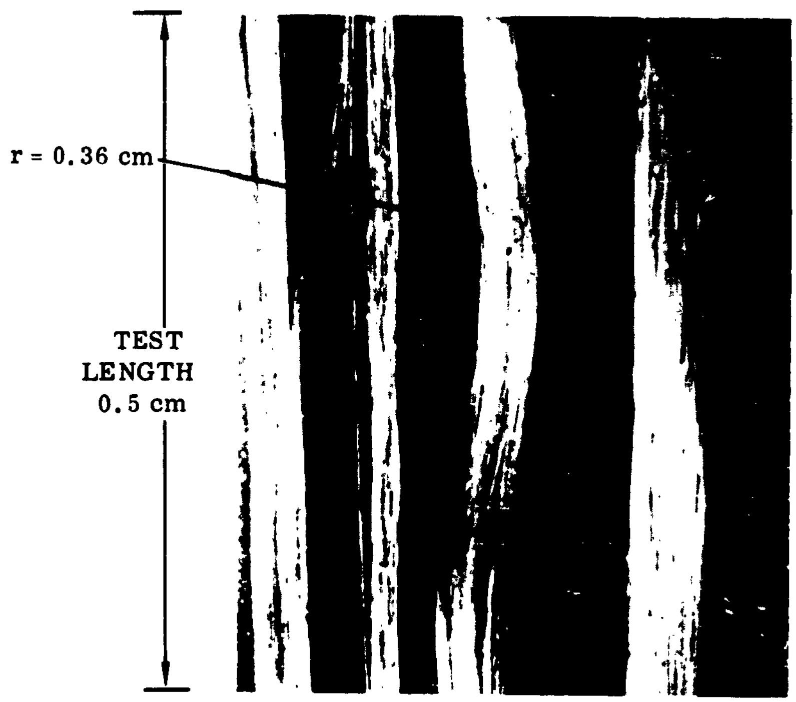

FIGURE A-6.

UNDULATIONS OF S-994 (9/19/62) STRAND DELIVERED ON CARDBOARD DRUM (20X)

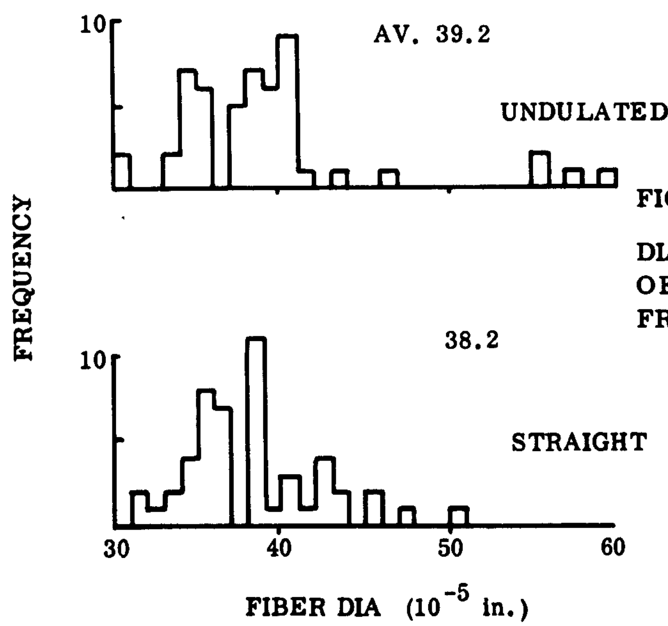

FIGURE A-7.

DIAMETER DISTRIBUTIONS OF TWO STRAND SECTIONS FROM S-994 (9/19/62)

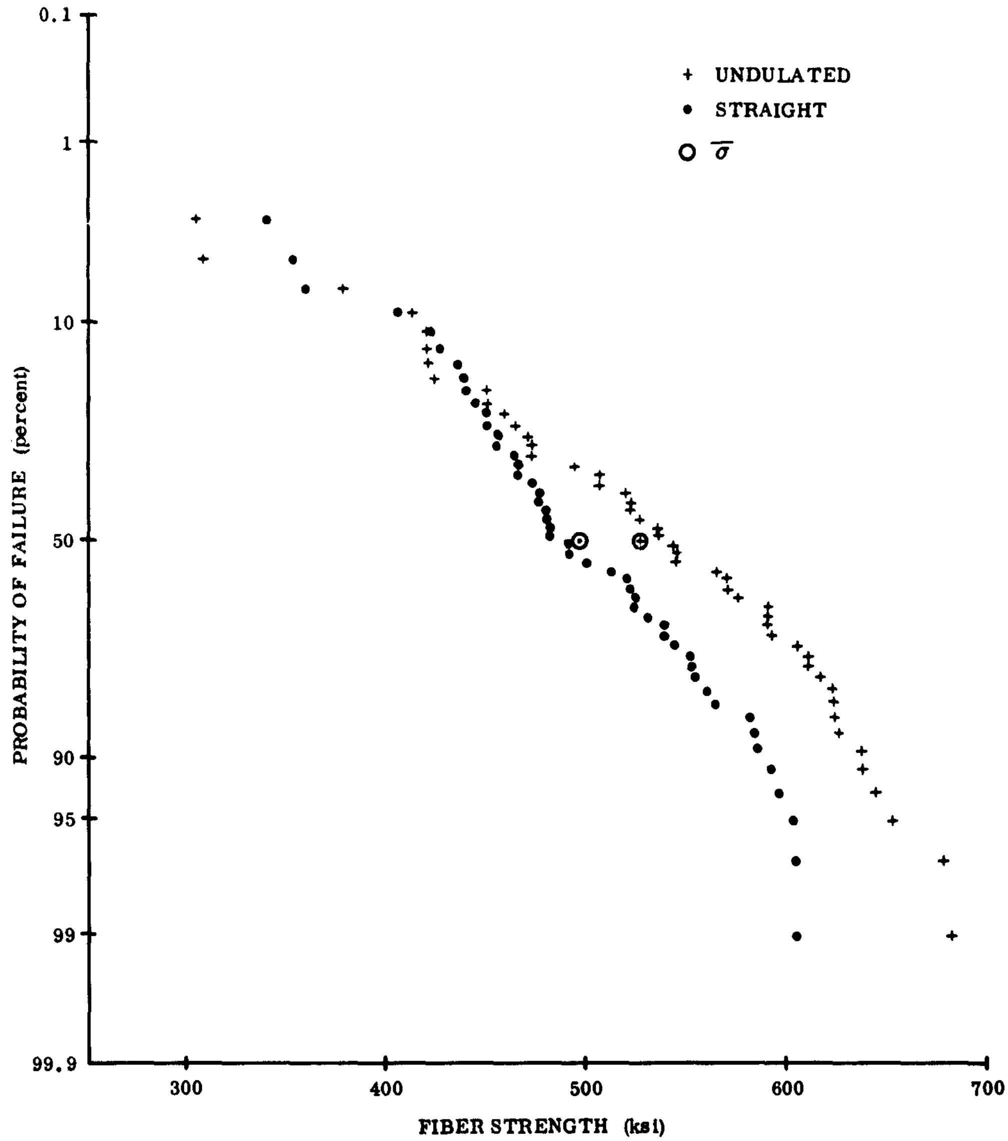

FIGURE A-8. FAILURE FREQUENCY DISTRIBUTIONS OF FIBERS FROM UNDULATED AND STRAIGHT S-994 (9-19-62) STRAND are plotted in Figure A-8. The undulation did not affect the fiber strength; in fact, the undulated section showed a 7 percent higher average (Table A-II) which lies, however, within strand strength scatter.

An interesting observation was made during short fiber tests on 994 fibers from strands. The organic binder (195) occasionally broke in a circumferential manner. As loading proceeded, the gap widened and the fiber usually failed within this gap. The process was most apparent on the excess coupling agent. Whether the chemical binder-glass interface was the site of separation could not be established.

A number of different gripping waxes were investigated. Most promising results were obtained with wax No. 3066 from Hi-Test Chemical Corporation, 722 - 64th Street, Brooklyn 20, New York. Slippage at short gage lengths (below 0.25 cm) of fibers separated from strand was reduced considerably. Slippage of uncoated fibers still remains a problem at gage lengths less than 1 cm although some improvement was achieved with the 3066.

In conclusion, average fiber strength fell within the X-994 envelope; the 0.025 cm point (not measured for X-994) is slightly higher. Undulation of strands of the magnitude observed does not seem to adversely affect fiber strength. There is indication that the amount of coupling agent plays a role in fiber strength preservation. Circumferential breakage of binder during tensile loading of fibers seems to cause fiber failure at that location.

APPENDIX B

SUMMARY OF AVERAGE STRENGTH DATA FROM FIBERS AND STRANDS

TABLE B-I

E-GLASS TEST DATA SUMMARY

	L (cm)	$\bar{D}$ (10^{-5} cm)	σ (ksi)	Standard Deviation (ksi)	Coefficient of Variation (%)	3rd Moment	4th Moment	No. of Samples	Sample Population	Remarks
Series I	1.5	37[1]	342	67.2	19.6	0.179-	2.268	25	612	Separated from three different strands
	6	37	284	33.8	12.0	0.050-	2.086	25		
	24	37	217	45.6	21.0	0.191-	1.977	25		
Series II 1	1.5	37[1]	292	57.8	19.8	0.008-	1.948	50	204	Separated from one strand H-1600
	3	37	277	40.1	14.5	0.157-	2.563	50		
	6	37	227	50.6	22.3	0.243	2.279	50		
	12	37	233	39.8	17.1	0.248-	3.235	50		
	24	37	227	45.3	20.0	0.697-	3.566	50		
Series II 2	1.5	37[1]	336	57.7	17.2	0.741-	2.866	29	204	Same as II 1 different strand section
	6	37	272	45.7	16.9	0.351-	2.443	29		
	12	37	237	46.8	19.7	0.192-	1.960	33		
	24	37	193	42.9	22.2	0.217-	2.673	29		
	30	37	164	18.8	11.5	0.409-	3.052	25		
Series II 1 and 2	1.5	37[1]	308	61.5	20.0	0.242-	2.831	79	204	Combined data
	3	37	277	40.1	14.5	0.157-	2.563	50		
	6	37	243	53.5	22.0	0.270-	2.444	79		
	12	37	234	42.7	18.2	0.246-	2.599	83		
	24	37	214	47.3	22.1	0.473-	3.486	79		
	30	37	164	18.8	11.5	0.409-	3.052	25		
U-Frames, HTS undamaged	3	45.0	421	20	4.7	--	--	8	single fibers	
HTS damaged	3	45.9	275	72	26.2	--	--	17		

[1] In accordance with information from Owens-Corning

TABLE B-II

994 GLASS FIBER TEST DATA SUMMARY

	L (cm)	D (10^{-5} in.)	σ (ksi)	Standard Deviation (ksi)	Coefficient of Variation (%)	3rd Moment	4th Moment	No. of Samples	Sample Population	Remarks
X-994 Series I	0.75	42.8	455	106.2	23.4	0.363-	2.96	60	65	
	1.5	43.3	435	95.5	22.0	0.034	2.99	60		
	3	43.5	388	107.1	27.6	0.428	3.51	60		
	6	42.0	346	108.1	31.2	0.172-	2.40	60		
	12	42.0	310	85.5	27.6	0.421-	3.42	43		
	24	42.0	325	80.0	24.6	0.555	2.21	60		
	24		369	94.6	25.6	0.804	3.53	25		Re-run different fibers.
X-994. Series II	0.05	41.1	572	53.2	9.3	0.366	3.36	13	30	
		40.9	590	45.8	7.8	0.153-	2.48	12		Slipped fibers omitted
	0.1	41.3	582	77.3	13.3	0.885	3.19	25		
	0.25	41.2	588	68.1	11.6	0.465	2.87	25		
	0.75	41.2	522	67.6	13.0	0.064	2.49	25		
	1.5	41.1	452	67.6	14.9	0.263-	2.63	25		
	3	41.1	450	36.3	8.1	0.198-	2.67	25		
X-994 Uncoated monofilament from cardboard drum	0.25	42.5	612	69.9	11.4	0.803-	3.48	25	continuous	Wound interspaced on cardboard drum
	0.5	40.6	515	100.2	19.5	0.852-	3.06	25		
	1.5	42.0	471	89.8	19.0	0.49	2.88	25	monofilament	
	12	42.6	419	105.3	25.1	0.33	2.96	20		
X-994. Virgin (U-Frames)										
Frame 1	0.5, 1.5	52.2	587	46.1	7.9	-	-	8	single fibers	Tested approximately two weeks after drawing
2	and 7 cm.	42.5	590	42.9	7.3	-	-	8		
3	See individual data	35.7	596	90.4	15.1	-	-	8		
4		45.1	526	65.7	11.6	-	-	8		
5		41.7	559	18.3	3.3		-	7		
6		41.4	541	41.9	7.7			8		
1-6		43.1	575	59.4	10.3	-	-	47		
1-3		43.7	619	18.0	2.9	-	-	19		
X-994 HTS. U-Frames										
(undamaged)	3	43.8	568	39	6.9	-	-	4	single fibers	
(damaged)	3	35.2	392	130	33.2	-		7		Accidental damage fibers touched each other
S-994 (July 1962)	0.025	38.4	557	71.1	12.8	0.501+	2.55	21	35	
	0.05	38.7	511	132.4	25.9	0.303-	2.46	23		
	0.1	38.2	422	101.3	24.0	1.139+	3.69	25		
	0.25	38.4	488	105.2	21.6	0.349+	2.20	25		
	0.5	38.1	561	95.2	17.0	0.421-	1.94	25		
	0.75	36.3	405	85.8	21.2	0.247-	2.95	25		
	1.5	36.5	272	52.2	19.2	0.159+	2.75	25		
	1.5	38.2	317	64.8	20.4	0.729-	2.79	25		
	3	36.4	428	89.3	20.9	0.549-	2.32	25		
	6	37.9	327	96.9	29.7	0.537-	2.55	25		
S-994 (July 1962) Fiber 1	0.5	34.5	588	104.0	17.7	0.946-	3.09	15	204	
2	0.5	34.5	518	90.1	17.4	1.000-	3.17	16		
3	0.5	36.0	516	95.8	18.6	1.357-	5.14	16		
4	0.5	36.0	501	65.6	13.1	0.428-	2.30	15		
5	0.5	36.0	619	110.3	17.8	0.572-	2.98	16		
6	0.5	36-43	531	137.5	25.9	0.458-	2.01	14		
7	0.5	39.0	547	97.9	17.9	0.072-	3.21	12		
8	0.5	39.0	507	100.4	19.8	0.369+	2.86	16		
9	0.5	39.0	478	144.4	30.2	0.888-	2.83	16		
10	0.5	39.0	486	130.5	26.8	0.694-	2.82	15		
1-10	0.5	37.2	528	41.2	7.8	1.057-	3.73	10		Average of averages
S-994 (9/19/62)	0.025	38.0	597	76.4	12.8	0.231-	2.19	23	45	
	0.05	38.5	579	70.0	12.1	0.397-	2.86	25		
	0.1	36.8	573	85.0	14.8	1.048-	4.56	25		
	0.25	38.0	526	92.8	17.7	0.656-	3.03	25		
	0.5	38.8	533	98.4	18.5	0.906-	4.44	25		
	0.75	38.2	404	101.6	25.1	0.149-	2.24	25		
	1.0	38.9	463	116.3	25.1	0.027-	2.15	25		
	1.5	37.1	452	134.7	29.8	0.339-	1.81	25		
	3	38.6	408	125.8	30.8	0.450-	2.38	25		
	6	36.9	364	90.3	24.8	0.813-	3.18	25		
Total	0.025	41.3	598	71.3	11.9	1.113-	4.72	25		Different section of strand
Group A	-	43.8	633	36.9	5.8	0.562+	2.19	15	15	
Group B		37.8	545	77.4	14.2	0.503-	3.27	10	10	
S-994 (9/19/62)										
Undulated	0.5	39.2	526	100.5	19.1	0.919-	3.86	50	204	Different sections of strand
Straight	0.5	38.2	491	76.9	15.6	0.971-	4.90	50	204	

TABLE B-III

X-994 AND E-GLASS STRAND TEST DATA SUMMARY

	X-994					E-Glass				
	Resin-Free		Resin			Resin-Free		Resin		
Test Length L (cm)	$\bar{\sigma}$ (ksi)	Number of Samples	$\bar{\sigma}$ (ksi)	Number of Samples	$\frac{\bar{\sigma}_R}{\bar{\sigma}_{R-F}}$	$\bar{\sigma}$ (ksi)	Number of Samples	$\bar{\sigma}$ (ksi)	Number of Samples	$\frac{\bar{\sigma}_R}{\bar{\sigma}_{R-F}}$
0.5								140	3	
0.75						270	7			
1.0			453	8						
1.5						221	4	225	2	1.0
2			441	9		223	5			
3	266	9	487	7	1.8	213	5	265	3	1.25
6	313	9	484	8	1.55	176	8	295	4	1.7
12	298	8	483	8	1.6	130	8	280	6	2.15
18	292	10	487	8	1.65	136	6			
24	281	8	439	10	1.55	123	7	290	4	2.35
34	259	9	409	9	1.55	130	5			

Note: Test environments were constant during the entire test period: 74-76°F and 49-55% RH.

APPENDIX C

INDIVIDUAL FIBER STRENGTH DATA

TABLE C-I

E-GLASS, SERIES I, II, 1 AND II, 2

INDIVIDUAL FIBER STRENGTH DATA, σ (ksi); AVERAGE DIAMETER,

Series I

	1.5 cm σ	6 cm σ	24 cm σ
1	197	224	126
2	224	231	141
3	281	231	155
4	281	240	169
5	281	240	169
6	288	267	175
7	288	267	190
8	288	267	196
9	308	267	196
10	308	274	196
11	308	274	203
12	329	274	210
13	345	281	217
14	365	281	217
15	372	295	224
16	379	295	246
17	386	208	246
18	386	210	253
19	393	310	253
20	393	315	274
21	399	315	274
22	422	315	274
23	442	337	274
24	443	337	274
25	456	344	287

Series II, 1

	1.5 cm σ	3 cm σ	6 cm σ	12 cm σ	24 cm σ
1	175	204	105	145	146
2	204	204	131	145	158
3	204	204	146	175	160
4	204	212	155	175	163
5	204	233	158	182	172
6	219	233	163	182	172
7	219	233	163	196	183
8	233	233	166	196	183
9	233	233	166	204	183
10	233	233	180	204	186
11	233	233	180	204	186
12	233	233	183	204	186
13	240	247	183	204	189
14	247	247	183	204	192
15	254	254	204	204	197
16	254	263	204	212	200
17	254	263	204	212	200
18	254	263	204	226	200
19	263	263	213	226	216
20	277	263	213	226	216
21	284	263	224	226	222
22	284	263	224	226	222
23	284	263	224	226	222
24	291	277	227	233	222
25	291	277	227	233	222
26	291	284	227	233	222
27	298	284	227	233	227
28	298	284	230	233	227
29	298	291	239	233	230
30	312	291	241	240	230
31	312	291	247	240	233
32	321	291	247	240	239
33	321	291	256	240	241
34	321	291	260	240	250
35	321	291	263	247	250
36	321	291	263	247	250
37	335	291	263	247	250
38	335	298	268	254	253
39	342	312	268	254	258
40	349	312	271	263	263
41	349	321	277	263	263
42	364	321	283	270	263
43	364	321	285	277	266
44	371	321	285	277	266
45	379	328	291	277	271
46	379	328	291	284	217
47	379	335	297	298	285
48	379	342	305	321	341
49	379	349	314	321	346
50	393	379	314	327	346

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TABLE C-I

E-GLASS, SERIES I, II, 1 AND II, 2

INDIVIDUAL FIBER STRENGTH DATA, σ (ksi); AVERAGE DIAMETER, $\bar{D} = 37 \times 10^{-5}$ in.

Series I

cm	6 cm σ	24 cm σ
07	224	126
24	231	141
31	231	155
31	240	169
31	240	169
38	267	175
08	267	190
38	267	196
08	267	196
08	274	196
08	274	203
29	274	210
45	281	217
55	281	217
72	295	224
79	295	246
06	208	246
36	210	253
03	310	253
03	315	274
09	315	274
22	315	274
42	337	274
43	337	274
56	344	287

Series II, 1

	1.5 cm σ	3 cm σ	6 cm σ	12 cm σ	24 cm σ
1	175	204	105	145	146
2	204	204	131	145	158
3	204	204	146	175	160
4	204	212	155	175	163
5	204	233	158	182	172
6	219	233	163	182	172
7	219	233	163	196	183
8	233	233	166	196	183
9	233	233	166	204	183
10	233	233	180	204	186
11	233	233	180	204	186
12	233	233	183	204	186
13	240	247	183	204	189
14	247	247	183	204	192
15	254	254	204	204	197
16	254	263	204	212	200
17	254	263	204	212	200
18	254	263	204	226	200
19	263	263	213	226	216
20	277	263	213	226	216
21	284	263	224	226	222
22	284	263	224	226	222
23	284	263	224	226	222
24	291	277	227	233	222
25	291	277	227	233	222
26	291	284	227	233	222
27	298	284	227	233	227
28	298	284	230	233	227
29	298	291	239	233	230
30	312	291	241	240	230
31	312	291	247	240	233
32	321	291	247	240	239
33	321	291	256	240	241
34	321	291	260	240	250
35	321	291	263	247	250
36	321	291	263	247	250
37	335	291	263	247	250
38	335	298	268	254	253
39	342	312	268	254	258
40	349	312	271	263	263
41	349	321	277	263	263
42	364	321	283	270	263
43	364	321	285	277	266
44	371	321	285	277	266
45	379	328	291	277	271
46	379	328	291	284	217
47	379	335	297	298	285
48	379	342	305	321	341
49	379	349	314	321	346
50	393	379	314	327	346

Series II, 2

	1.5 cm σ	6 cm σ	12 cm σ	24 cm σ	30 cm σ
1	239	170	158	108	119
2	263	185	170	128	128
3	271	206	175	133	135
4	280	222	175	141	146
5	280	222	183	142	153
6	288	222	185	155	155
7	290	229	185	159	155
8	292	239	195	166	156
9	298	244	203	170	159
10	301	253	206	175	160
11	311	260	207	179	160
12	312	273	209	183	160
13	312	274	210	185	166
14	314	275	217	186	168
15	320	278	222	190	168
16	328	281	236	192	169
17	334	281	237	199	169
18	341	288	246	200	172
19	348	290	251	200	176
20	355	291	253	216	178
21	356	305	256	222	179
22	359	310	258	224	185
23	362	316	258	224	193
24	385	312	263	226	193
25	419	314	263	233	197
26	419	325	266	234	-
27	433	335	273	258	
28	444	337	304	278	
29	479	355	308	287	
30	-	-	312	-	
31			312		
32			315		
33			317		

TABLE C-II

X-994, SERIES I

INDIVIDUAL FIBER STRENGTH DATA, σ (ksi);

A (10^{-6} in.^{2}) AVERAGE CROSS-SECTIONAL AREA

$\bar{A}$ =	0.144	0.147	0.149	0.138	0.138	0.138	Re-run
Fiber No.	0.75 cm σ	1.5 cm σ	3 cm σ	6 cm σ	12 cm σ	24 cm σ	24 cm σ
1	294	608	345	473	343	359	110
2	385	458	357	391	359	462	187
3	449	380	274	151	467	272	260
4	483	336	387	349	346	349	266
5	497	563	227	380	371	123	280
6	544	629	452	447	304	344	323
7	452	458	409	223	417	422	340
8	421	333	480	363	262	275	341
9	448	279	392	538	232	342	343
10	581	537	444	580	294	373	346
11	526	542	430	377	339	348	355
12	556	511	232	496	346	375	357
13	452	518	426	503	291	447	374
14	332	401	438	342	294	385	374
15	521	429	395	286	192	320	401
16	399	565	367	396	538	272	412
17	399	568	317	414	327	323	421
18	531	468	473	317	141	275	434
19	497	408	395	343	183	151	445
20	641	399	292	219	314	396	450
21	462	372	301	580	215	247	456
22	392	458	310	441	416	319	461
23	400	505	438	296	518	406	471
24	556	386	445	293	257	216	491
25	325	250	486	357	295	288	525
26	406	501	671	439	234	343	
27	319	221	367	238	317	404	
28	527	544	501	299	351	293	
29	625	372	351	367	278	401	
30	556	279	176	489	351	295	
31	420	269	381	501	397	514	
32	676	420	480	356	174	391	
33	406	501	480	126	378	355	
34	406	415	501	308	282	317	
35	323	673	405	224	320	299	
36	182	322	120	293	327	328	
37	194	430	515	328	274	385	
38	337	405	409	251	175	308	
39	556	448	493	341	192	201	
40	467	445	295	263	324	265	
41	299	501	544	224	250	257	
42	542	329	342	165	330	396	
43	533	422	119	313	327	364	
44	580	358	233	342	-	436	
45	474	401	444	293	-	257	
46	399	437	444	251	-	144	
47	528	408	497	158	-	370	
48	458	444	399	349	-	337	
49	384	437	438	475	-	226	
50	583	494	283	364	-	288	

TABLE C-III

X-994, Series II

Individual Fiber Strength Data, σ (ksi); Diameter, D(10^{-5} in.)

Fiber No.	0.05 cm		0.1 cm		0.25 cm		0.75 cm		1.5 cm		3 cm	
	σ	D	σ	D	σ	D	σ	D	σ	D	σ	D
1	581*	41.4	613	41.1	568	41.8	491	43.6	433	41.4	432	40.0
2	680	41.4	530	41.1	499	42.9	371	40.7	434	42.5	415	41.8
3	502*	40.7	616	42.9	526	42.5	518	40.0	472	41.4	396	36.8
4	547*	40.4	663	38.2	431	41.8	483	41.1	514	41.4	449	41.4
5	544*	39.6	593	43.1	486	37.9	644	40.4	472	37.9	472	41.4
6	632	40.4	660	42.1	622	38.2	632	40.4	468	42.9	469	43.2
7	594	37.5	636	41.8	457	40.0	484	41.4	553	41.4	430	37.9
8	571	40.4	639	43.2	548	42.1	518	37.1	417	41.4	501	43.6
9	587*	41.4	528	37.9	610	41.4	518	41.4	454	41.8	400	43.2
10	650*	43.2	564	43.6	657	41.4	417	39.6	618	42.9	425	37.9
11	613*	41.8	691	40.0	575	43.2	522	43.2	499	41.8	380	42.1
12	601	41.1	586	43.2	660	43.2	522	43.6	426	41.4	464	42.9
13	622	41.4	632	39.6	650	43.2	565	37.5	342	39.6	497	42.9
14	553*	41.4	438	42.5	704	41.8	562	42.9	601	42.1	410	42.9
15	568*	41.8	564	41.4	638	38.6	604	42.1	455	36.8	503	42.1
16	541	41.4	543	43.6	576	41.4	479	43.2	490	43.2	449	42.5
17	553	41.4	533	41.8	610	41.4	579	36.0	397	43.6	454	41.8
18	507	41.4	406	38.2	626	42.9	533	43.2	511	41.8	454	41.8
19	599*	41.4	403	37.9	628	43.2	575	43.2	395	40.0	432	40.0
20	431*	41.8	680	35.0	579	41.8	583	42.9	366	36.0	483	37.9
21	590	41.4	646	42.5	580	40.0	449	43.6	361	42.5	461	41.8
22	533	41.4	618	43.2	571	40.4	464	42.9	507	41.4	533	41.8
23	507*	41.4	668	41.4	594	42.9	634	41.8	427	42.5	458	43.2
24	634	41.4	553	43.6	710	34.3	469	40.0	406	38.2	417	37.9
25	526*	40.7	584	43.6	596	43.2	444	40.0	360	43.9	458	37.9

*Rupture outside nominal gage length.

TABLE C-IV

X-994, UNCOATED MONOFILAMENT FROM CARDBOARD DRUM

INDIVIDUAL FIBER STRENGTH DATA, σ (ksi); DIAMETER, D (10^{-5} in.)

	0.25 cm		0.5 cm		1.5 cm		12 cm	
	σ	D	σ	D	σ	D	σ	D
1	416	41.4	245	421.	245	42.4	151	42.8
2	496	41.4	363	41.7	324	42.1	259	43.5
3	522	43.5	368	38.5	344	42.1	285	40.3
4	549	43.5	380	38.2	360	42.4	324	43.5
5	554	41.4	391	39.6	408	40.3	330	43.5
6	566	41.4	446	42.1	408	41.4	367	42.8
7	574	43.5	466	38.5	412	41.7	377	40.3
8	577	41.4	480	42.1	433	43.2	388	42.8
9	589	41.4	483	41.7	440	42.8	418	43.5
10	594	43.2	504	39.6	450	39.9	418	43.5
11	600	41.4	513	42.1	461	39.2	456	42.8
12	605	43.5	520	41.4	471	43.2	475	42.8
13	616	43.5	530	39.6	476	42.1	480	43.2
14	626	43.5	546	38.5	492	41.4	480	43.5
15	647	41.4	567	38.5	496	43.2	487	40.3
16	647	41.4	580	42.1	515	41.7	499	43.2
17	647	43.5	586	38.5	520	43.2	504	41.7
18	658	41.4	592	38.2	521	41.7	516	42.4
19	668	43.5	597	38.2	539	43.2	552	43.2
20	679	43.2	600	42.1	555	41.4	572	40.3
21	681	41.4	600	41.4	555	41.5	-	-
22	689	43.5	609	42.1	564	43.2	-	-
23	700	43.5	620	42.1	576	41.7	-	-
24	700	43.5	636	41.7	606	43.2	-	-
25	710	43.5	647	42.1	618	41.4	-	-

TABLE C-V

X-994 VIRGIN, (U-FRAMES)

INDIVIDUAL FIBER STRENGTH DATA, σ (ksi); AND DIAMETERS D (10^{-5} in.)

Frame	Fiber	σ	D	Test Length (cm)
I	1	492	52.2	7
	2	496	52.2	
	3	538	52.2	
	4	620	52.2	
	5	620	52.2	
	6	580	52.2	
	7	620	52.2	
	8	634	52.2	
II	1	545	44.3	7
	2	640	41.8	
	3	619	42.1	
	4	573	43.9	
	5	505	41.8	1.5
	6	615	42.1	
	7	602	43.2	
	8	623	42.5	
III	1	357	35.7	1.5
	2	624	35.7	
	3	629	35.7	
	4	643	35.7	
	5	629	35.7	7
	6	624	35.7	
	7	629	35.7	
	8	630	35.7	

Frame	Fiber	σ	D	Test Length (cm)
IV	1	400	46.0	7
	2	592	45.4	
	3	618	44.3	
	4	599	45.0	
	5	599	45.0	1.5
	6	597	45.0	
	7	600	45.0	
	8	599	45.0	
V	1	531	43.2	1.5
	2	579	42.1	
	3	568	42.1	
	4	574	41.4	0.5
	5	574	41.4	
	6	534	41.4	
	7	555	41.4	
	8	-	--	-
VI	1	563	41.4	0.5
	2	578	41.4	
	3	460	41.4	
	4	553	41.1	
	5	554	41.4	
	6	565	41.8	1.5
	7	481	41.4	
	8	575	41.4	

TABLE C-VI

X-994 HTS, E GLASS HTS, U-FRAMES

INDIVIDUAL FIBER STRENGTH DATA, σ (ksi); DIAMETER D (10^{-5} in.)

	E		
	σ	D	
1	423	42.1	un-damaged
2	430	41.8	
3	435	46.1	
4	389	46.1	
5	400	45.7	
6	449	45.7	
7	404	46.1	
8	436	45.7	
9	272	42.1	damaged
10	222	52.1	
11	250	53.2	
12	164	52.1	
13	273	53.8	
14	358	43.2	
15	118	42.5	
16	311	42.5	
17	261	42.1	
18	330	44.6	
19	276	46.8	
20	263	40.4	
21	420	50.4	
22	280	42.1	
23	212	43.6	
24	216	39.6	
25	322	48.5	

Note: Test length 3 cm.

	X-994		
	σ	D	
1	532	44.3	un-damaged
2	516	43.5	
3	608	43.5	
4	594	43.5	
5	609	32.5	
6	318	32.2	damaged
7	185	33.9	
8	442	33.9	
9	403	36.0	
10	340	36.8	
11	440	40.3	

Note: "Damage" was caused by loose fibers on U-Frames touching mounted fibers* and being removed.

*(Not necessarily in test length sections).

TABLE C-VII

S-994 (JULY 62)

INDIVIDUAL FIBER STRENGTH DATA, σ (ksi); DIAMETE

	0.025 cm		0.05 cm		0.1 cm		0.25 cm		0.5 cm		0.75 cm		
	σ	D	σ	D	σ	D	σ	D	σ	D	σ	D	σ
1	434	41.0	222	69.9	272	39.9	306	38.5	378	33.9	219	34.2	14
2	453	42.8	293	44.9	295	35.7	341	35.7	388	36.4	241	33.9	21
3	484	41.4	310	35.7	328	40.3	345	42.8	431	38.9	303	34.2	22
4	499	44.9	372	35.7	341	35.7	377	42.8	435	35.7	341	35.7	22
5	499	44.9	399	38.5	341	35.7	388	42.8	450	35.7	346	41.4	23
6	505	34.2	413	40.3	341	35.7	403	35.7	461	41.4	354	34.2	23
7	528	35.7	426	38.5	345	37.8	422	37.4	461	41.4	387	34.2	23
8	528	35.7	486	40.3	362	44.9	427	41.4	498	43.2	388	42.8	23
9	528	44.9	488	34.2	371	39.9	434	38.2	517	42.8	398	39.2	23
10	539	34.2	497	35.7	379	42.1	443	45.3	544	39.9	404	34.2	23
11	543	35.7	497	35.7	388	35.7	447	33.9	559	38.5	404	34.2	25
12	543	35.7	528	35.7	392	41.4	466	35.7	569	38.2	407	38.2	25
13	557	42.1	528	35.7	404	41.4	470	37.8	588	41.4	419	35.7	25
14	559	35.7	528	42.8	404	41.4	473	41.4	606	34.2	421	34.2	26
15	577	41.4	552	39.2	413	33.9	497	37.8	621	35.7	450	35.7	27
16	605	39.6	570	35.3	414	37.8	502	38.2	621	35.7	458	39.9	27
17	609	36.0	590	34.2	439	38.5	512	35.7	625	42.4	464	33.9	28
18	628	33.2	605	35.7	450	41.4	512	35.7	626	44.2	465	37.4	28
19	679	32.8	629	39.2	456	37.8	565	41.4	636	35.7	519	38.5	31
20	691	37.8	683	35.7	461	41.4	586	38.5	640	34.2	556	34.2	32
21	700	35.7	691	34.2	472	34.2	634	41.0	644	39.9	559	35.7	33
22	-	-	700	35.7	546	30.7	636	37.8	652	35.7	-	-	33
23	-	-	730	35.7	587	44.9	640	34.2	652	35.7	-	-	34
24	-	-	-	-	666	31.7	658	41.7	683	35.7	-	-	35
25	-	-	-	-	674	34.2	700	35.7	714	35.7	-	-	37

*In sequence.

**End of strand.

TABLE C-VII

S-994 (JULY 62)

INDIVIDUAL FIBER STRENGTH DATA, σ (ksi); DIAMETER, D $(10^{-5}$ in.)

0.1 cm		0.25 cm		0.5 cm		0.75 cm		1.5 cm*		1.5 cm**		3 0 cm		6.0 cm	
σ	D	σ	D	σ	D	σ	D	σ	D	σ	D	σ	D	σ	D
272	39.9	306	38.5	378	33.9	219	34.2	149	38.2	172	42.8	246	35.7	94	43.2
295	35.7	341	35.7	388	36.4	241	33.9	217	35.7	191	40.7	250	33.5	154	41.4
328	40.3	345	42.8	431	38.9	303	34.2	223	33.9	202	34.2	278	34.2	205	41.4
341	35.7	377	42.8	435	35.7	341	35.7	226	38.5	222	43.2	296	35.7	214	34.2
341	35.7	388	42.8	450	35.7	346	41.4	236	34.2	258	42.8	355	35.7	220	39.9
341	35.7	403	35.7	461	41.4	354	34.2	236	34.2	269	42.8	374	41.7	228	38.5
345	37.8	422	37.4	461	41.4	387	34.2	236	34.2	269	34.2	383	42.8	283	43.2
362	44.9	427	41.4	498	43.2	388	42.8	236	34.2	286	34.2	389	38.5	296	43.2
371	39.9	434	38.2	517	42.8	398	39.2	236	34.2	318	43.2	396	34.2	297	34.9
379	42.1	443	45.3	544	39.9	404	34.2	237	42.8	333	39.2	396	34.2	299	34.2
388	35.7	447	33.9	559	38.5	404	34.2	252	34.2	337	34.2	406	34.2	304	38.5
392	41.4	466	35.7	569	38.2	407	38.2	258	42.8	337	34.2	428	34.2	331	34.2
404	41.4	470	37.8	588	41.4	419	35.7	258	42.8	337	34.2	449	44.2	337	41.4
404	41.4	473	41.4	606	34.2	421	34.2	264	35.7	340	41.7	459	42.8	341	44.9
413	33.9	497	37.8	621	35.7	450	35.7	275	34.9	341	35.7	459	33.9	343	46.0
414	37.8	502	38.2	621	35.7	458	39.9	277	39.6	345	42.1	463	35.3	374	34.2
439	38.5	512	35.7	625	42.4	464	33.9	286	34.2	348	35.3	490	38.5	394	35.7
450	41.4	512	35.7	626	44.2	465	37.4	286	34.2	359	38.5	499	38.5	396	36.0
456	37.8	565	41.4	636	35.7	519	38.5	313	34.6	360	43.2	503	34.2	406	36.0
461	41.4	586	38.5	640	34.2	556	34.2	329	42.4	365	39.6	513	34.9	417	34.2
472	34.2	634	41.0	644	39.9	559	35.7	337	34.2	370	34.2	524	34.2	434	35.7
546	30.7	636	37.8	652	35.7	-	-	337	32.2	372	35.7	535	34.2	438	34.2
587	44.9	640	34.2	652	35.7	-	-	340	38.9	372	35.7	535	34.2	438	34.2
666	31.7	658	41.7	683	35.7	-	-	357	35.7	373	37.8	535	34.2	449	34.2
674	34.2	700	35.7	714	35.7	-	-	378	33.9	438	34.2	535	34.2	464	32.4

TABLE C-VIII.

S-994 (JULY 1962) LONG FIBER STRENGTH SURVEY.

(Tested at 16 Locations, Constant Test Length, 0.5 cm)

Fiber Number	1	2	3	4	5	6	7	8	9	10	Average Strength (ksi)	Standard Deviation (ksi)	Coefficient of Variation (%)
Diameter x 10^{-5} inch	34.5	34.5	36	36	36	36 to 43	39	39	39	39			
Test Location	ksi												
1	507	337	233	482	573	518	538	390	186	452	422	125	30
2	642	507	630	513	492	434	558	598	505	Lost	542	66	12
3	675	642	528	544	540	617	598	425	557	398	552	84	15
4	557	573	467	425	354	701	619	730	333	452	521	130	25
5	423	557	575	528	709	Lost	752	452	570	214	531	150	28
6	625	575	482	373	624	Lost	532	465	390	530	511	86	17
7	642	315	560	497	591	703	400	505	570	530	531	107	20
8	709	523	452	Lost	675	556	558	502	402	605	554	93	17
9	557	540	544	467	591	682	359	612	525	690	557	93	17
10	709	624	542	482	492	542	571	456	453	640	551	81	15
11	685	405	591	389	775	435	479	531	639	542	547	120	22
12	507	492	560	575	685	636	598	527	492	234	531	116	22
13	608	557	560	497	642	404	Lost	650	645	552	568	76	13
14	642	591	435	590	775	630	Lost	531	645	502	593	92	16
15	Lost	507	435	592	790	288	Lost	425	167	568	467	170	37
16	337	540	645	560	636	285	Lost	318	570	386	475	135	28
Average Strength (ksi)	588	518	516	501	619	531	547	507	478	486	528		
Standard Deviation (ksi)	104	90	96	66	110	138	98	100	144	130	41		
Coefficient of Variation (%)	17.7	17.4	18.6	13.1	18	26	18	19.8	30.2	26.8	8		

TABLE C-IX

S-994 (9/19/62)

INDIVIDUAL FIBER STRENGTH DATA. σ (ksi); DIAMETER, D (10^{-5} in.)

	0.025 cm[2]		0.025 cm		0.05 cm		0.1 cm		0.25 cm		0.5 cm		0.75 cm	
	σ	D	σ	D	σ	D	σ	D	σ	D	σ	D	σ	D
1	379	42.1	440	44.9	418	38.9	311	35.7	285	38.2	235	39.9	200	38.
2	466	35.7	466	35.7	446	39.9	426	36.0	378	39.6	357	42.1	218	35.
3	499	40.3	502	38.2	472	36.0	487	36.0	388	35.7	454	39.6	280	38.
4	559	35.7	513	35.7	497	35.7	493	36.4	409	39.9	466	35.7	290	39.
5	559	35.7	514	32.8	528	35.7	513	35.7	435	35.7	466	35.7	318	37.
6	559	35.7	544	35.7	531	43.2	519	38.5	473	41.4	474	37.1	318	37.
7	575	35.7	550	37.4	552	39.2	533	39.9	482	35.7	482	35.7	326	35.
8	580	39.6	560	40.3	559	35.7	543	38.2	488	37.1	487	40.3	328	39.
9	587[1]	44.9	572	40.3	559	38.5	559	35.7	492	39.6	499	40.3	373	35.
10	588[1]	45.3	575	35.7	572	42.8	559	38.5	498	37.8	519	38.5	373	38.
11	591	42.1	577	41.4	577	41.4	567	37.8	508	41.4	521	37.4	389	40.
12	597	44.6	611	33.2	580	39.6	579	36.0	523	34.2	523	36.4	400	38.
13	605	39.6	621	35.7	590	34.2	581	37.8	546	38.5	523	40.3	404	32.
14	607[1]	44.9	621	35.7	590	35.7	582	36.4	555	39.6	523	40.3	404	35.
15	607[1]	44.9	625	46.7	593	39.6	589	37.1	567	39.6	542	39.6	424	42.
16	617[1]	44.9	647	33.2	597	36.4	594	36.0	575	35.7	581	37.8	449	43.
17	625[1]	42.1	657	40.3	599	38.5	597	36.4	586	38.5	584	40.3	450	40.
18	643[1]	39.6	664	37.8	606	35.7	609	30.7	599	38.5	590	35.7	453	38.
19	643[1]	46.4	668	35.7	613	38.5	645	40.3	600	41.4	607	44.9	478	36.
20	646[1]	44.9	669	42.1	613	38.5	657	40.3	600	36.7	615	42.8	504	39.
21	665[1]	44.9	693	41.4	656	39.6	662	37.4	606	35.7	621	35.7	517	37.
22	670[1]	41.4	706	38.5	670	41.4	666	38.5	607	34.2	637	35.7	531	37.
23	684	35.7	730	35.7	681	39.9	668	35.7	618	39.6	639	38.5	538	36.
24	700[1]	43.2	-	--	691	37.8	675	32.4	636	37.8	681	39.9	559	38.
25	705	44.9	-	--	693	38.9	700	35.7	691	37.8	691	37.8	580	39.

[1] Group A

[2] Different Section of Strand

TABLE C-IX

S-994 (9/19/62)

INDIVIDUAL FIBER STRENGTH DATA. σ (ksi);

DIAMETER, D (10^{-5} in.)

5 cm	0.1 cm		0.25 cm		0.5 cm		0.75 cm		1 0 cm		1.5 cm		3 cm		6 cm	
D	σ	D	σ	D	σ	D	σ	D	σ	D	σ	D	σ	D	σ	D
38.9	311	35.7	285	38.2	235	39.9	200	38.5	263	37.8	196	41.4	128	39.2	138	37.8
39.9	426	36.0	378	39.6	357	42.1	218	35.7	266	38.5	249	35 7	202	35.7	186	35.7
36.0	487	36.0	388	35.7	454	39.6	280	38.5	293	38.5	264	35.7	216	42.8	200	38.5
35.7	493	36.4	409	39.9	466	35.7	290	39.6	311	35.7	280	38.5	240	38.5	277	37.8
35.7	513	35.7	435	35.7	466	35.7	318	37.8	342	35.7	290	37.8	277	37.8	293	38.5
43.2	519	38.5	473	41.4	474	37.1	3.8	37.8	367	42.8	295	35.7	277	39.6	311	35.7
39.2	533	39.9	482	35.7	482	35.7	326	35.7	373	35.7	295	35.7	346	37.8	337	34.2
35.7	543	38.2	488	37.1	487	40.3	328	39.6	388	35.7	357	35.7	369	47.4	340	38.9
38.5	559	35.7	492	39.6	499	40 3	373	35.7	401	37.8	401	37 8	373	35.7	342	35.7
42.8	559	38.5	498	37.8	519	38 5	373	38.5	446	39.9	413	38.5	387	37.8	346	37.8
41.4	567	37 8	508	41.4	521	37.4	389	40.3	456	37.8	467	39.6	404	35.7	346	38.5
39.6	579	36.0	523	34.2	523	36.4	400	38.5	462	39.2	482	35.7	413	38.5	373	35.7
34.2	581	37.8	546	38.5	523	40.3	404	32.8	467	43.2	482	35.7	429	37.8	373	36.4
35.7	582	36.4	555	39.6	523	40.3	404	35.7	475	40.3	497	35.7	442	37.8	386	38.5
39.6	589	37.1	567	39.6	542	39.6	424	42.1	496	41.4	508	36.4	442	37.8	404	35.7
36.4	594	36.0	575	35.7	581	37.8	449	43.5	511	40.3	517	39.6	454	39.6	405	34.2
38.5	597	36.4	586	38.5	584	40.3	450	40.3	513	35.7	539	37.8	466	38.5	415	37.8
35.7	609	30.7	599	38.5	590	35.7	453	38.5	533	38.5	559	35.7	479	39.6	416	39.6
38.5	645	40.3	600	41.4	607	44.9	478	36.4	539	37.8	567	37.8	493	38.5	434	38.2
38.5	657	40.3	600	36.7	615	42.8	504	39.6	549	38.9	575	35.7	528	35.7	438	34.2
39.6	662	37.4	606	35.7	621	35.7	517	37.1	568	44.9	580	39.6	528	35.7	442	37.8
41.4	666	38.5	607	34.2	637	35.7	531	37.1	591	42.1	594	36.0	554	41.4	453	38.5
39.9	668	35.7	618	39.6	639	38.5	538	36.4	637	35.7	606	35.7	584	43.2	466	35.7
37.8	675	32.4	636	37.8	681	39.9	559	38.5	668	39.2	621	35.7	590	35.7	466	35.7
38.9	700	35.7	691	37.8	691	37.8	580	39.6	668	39.2	666	38.5	590	35.7	512	35.7

TABLE C-X

S-994 (9/19/62), UNDULATED AND STRAIGHT SECTION

INDIVIDUAL FIBER STRENGTH DATA, (ksi); AND DIAMETER D (10^{-5} in.) TEST LENGTH 0.5 cm

	Undulated			Straight	
	σ	D		σ	D
1	575	34.3	1	438	38.6
2	568	55.7	2	455	42.1
3	459	55.7	3	554	33.2
4	592	40.0	4	585	38.6
5	494	40.0	5	524	34.3
6	682	39.3	6	340	40.4
7	637	38.6	7	596	43.2
8	625	37.9	8	464	38.6
9	590	35.7	9	568	38.2
10	609	34.3	10	544	38.6
11	417	37.9	11	482	35.7
12	644	33.2	12	584	38.6
13	608	30.7	13	518	40.0
14	507	34.3	14	359	38.6
15	507	34.3	15	466	35.7
16	590	34.3	16	553	36.4
17	622	35.7	17	435	35.7
18	622	35.7	18	479	38.6
19	590	35.7	19	604	42.1
20	622	35.7	20	212	36.4
21	464	33.2	21	443	39.6
22	420	30.7	22	473	34.3
23	424	39.3	23	479	38.6
24	308	40.0	24	538	36.4
25	378	38.2	25	438	38.6
26	636	37.9	26	421	40.0
27	412	38.6	27	500	50.7
28	536	37.9	28	604	42.1
29	603	39.3	29	490	43.6
30	616	39.3	30	524	36.4
31	545	35.7	31	538	36.4
32	652	37.9	32	456	31.0
33	473	41.4	33	532	38.6
34	305	38.6	34	450	41.4
35	449	40.4	35	522	47.9
36	543	43.2	36	352	45.0
37	201	34.3	37	477	31.0
38	518	38.6	38	607	45.0
39	545	38.6	39	476	42.9
40	567	40.0	40	560	35.7
41	522	40.4	41	592	34.3
42	570	46.1	42	449	36.4
43	471	59.5	43	552	35.0
44	419	57.5	44	512	32.9
45	473	34.3	45	482	35.7
46	535	40.4	46	426	36.8
47	522	40.4	47	467	35.7
48	677	38.6	48	491	34.3
49	449	40.4	49	582	33.9
50	527	39.3	50	405	35.7

TABLE C-XI

MAXIMUM AND MINIMUM FIBER STRENGTH (ksi) OF E AND 994 GLASS AT DIFFE

MAXIMUM

L (cm)	E Series I	E Series II	X-994 Series I	X-994 Series II	X-994 Monofil	X-994 Virgin (U-Frames)	S-994 (July 1962)	X-994 (July 1962) Long Fiber Survey	S (9/
0.025							699 (21)		705 730
0.05				680 (25)			730 (23)		693
0.1				691 (25)			674 (25)		699
0.25				710 (25)	710 (25)		699 (25)		691
0.5					647 (25)	578 (9)	714 (25)	790 (150)	691
0.75			676 (50)	644 (25)	618 (25)		559 (21)		580
1.0									668
1.5	456 (25)[1]	479 (79)	673 (50)	618 (25)	618 (25)	643 (18)	378 (25) 438 (25)		688
3		379 (50)	671 (50)	533 (25)			535 (25)		590
6	344 (25)	355 (79)	580 (50)			640 (20)	464 (25)		51
12		327 (83)	538 (43)		572 (20)				
24	287 (25)	346 (79)	514 (50) 525 (25)						
30		197 (25)							

MINIMUM

L (cm)	E Series I	E Series II	X-994 Series I	X-994 Series II	X-994 Monofil	X-994 Virgin (U-Frames)	S-994 (July 1962)	X-994 (July 1962) Long Fiber Survey	S (9/
0.025							434 (21)		37 44
0.05				507 (12)			222 (23)		41
0.1				403 (25)			272 (25)		31
0.25				431 (25)	416 (25)		306 (25)		28
0.5					245 (25)	460 (9)	378 (25)	167 (150)	23
0.75			182 (50)	371 (25)			213 (21)		20
1.0									26
1.5	197 (25)[1]	175 (79)	221 (50)	342 (25)	245 (25)	357 (18)	149 (25) 172 (25)		19
3		204 (50)	119 (50)	380 (25)			246 (25)		12
6	224 (25)	105 (79)	126 (50)			400 (20)	94 (25)		13
12		145 (83)	141 (43)		151 (20)				
24	126 (25)	108 (79)	123 (50) 110 (25)						
30		119 (25)							

[1]Sample Size	Probability of Failure Max.	Min.
(25)	98%	2%
(50)	99%	1%
(80)	99.4%	0.6%
(150)	99.7%	0.3%

TABLE C-XI

…XIMUM AND MINIMUM FIBER STRENGTH (ksi) OF E AND 994 GLASS AT DIFFERENT TEST LENGTHS

MAXIMUM

…E …es I	E Series II	X-994 Series I	X-994 Series II	X-994 Monofil	X-994 Virgin (U-Frames)	S-994 (July 1962)	X-994 (July 1962) Long Fiber Survey	S-994 (9/19/62)	S-994 (9/19/62) Undulated and Straight	E and X-994 HTS (U-Frames) Undamaged and Damaged: E	E and X-994 HTS (U-Frames) Undamaged and Damaged: X-994
						699 (21)		705 (25) 730 (23)			
			680 (25)			730 (23)		693 (25)			
			691 (25)			674 (25)		699 (25)			
			710 (25)	710 (25)		699 (25)		691 (25)			
				647 (25)	578 (9)	714 (25)	790 (150)	691 (25)	682 (50) 607 (50)		
		676 (50)	644 (25)	618 (25)		559 (21)		580 (25)			
								668 (25)			
(25)[1]	479 (79)	673 (50)	618 (25)	618 (25)	643 (18)	378 (25) 438 (25)		688 (25)			
	379 (50)	671 (50)	533 (25)			535 (25)		590 (25)		449 (8) 420 (17)	609 (5) 442 (6)
(25)	355 (79)	580 (50)			640 (20)	464 (25)		512 (25)			
	327 (83)	538 (43)		572 (20)							
(25)	346 (79)	514 (50) 525 (25)									
	197 (25)										

MINIMUM

…E …es I	E Series II	X-994 Series I	X-994 Series II	X-994 Monofil	X-994 Virgin (U-Frames)	S-994 (July 1962)	X-994 (July 1962) Long Fiber Survey	S-994 (9/19/62)	S-994 (9/19/62) Undulated and Straight	E and X-994 HTS (U-Frames) Undamaged and Damaged: E	E and X-994 HTS (U-Frames) Undamaged and Damaged: X-994
						434 (21)		379 (25) 440 (23)			
			507 (12)			222 (23)		418 (25)			
			403 (25)			272 (25)		311 (25)			
			431 (25)	416 (25)		306 (25)		285 (25)			
				245 (25)	460 (9)	378 (25)	167 (150)	235 (25)	201 (50) 212 (50)		
		182 (50)	371 (25)			219 (21)		200 (25)			
								263 (25)			
(25)[1]	175 (79)	221 (50)	342 (25)	245 (25)	357 (18)	149 (25) 172 (25)		196 (25)			
	204 (50)	119 (50)	380 (25)			246 (25)		128 (25)		389 (8) 118 (17)	516 (5) 185 (6)
(25)	105 (79)	126 (50)			400 (20)	94 (25)		138 (25)			
	145 (83)	141 (43)		151 (20)							
(25)	108 (79)	123 (50) 110 (25)									
	119 (25)										

Probability of Failure

Max.	Min.
98%	2%
99%	1%
99.4%	0.6%
99.7%	0.3%

2

APPENDIX D

ABSTRACT OF THEORETICAL ANALYSIS OF BI-MODAL FAILURE

Figure

A theoretical analysis of the bimodal failure distribution has been undertaken. Since one mode of failure (for long fiber lengths) follows a linear relationship when plotted on log-log paper, an attempt has been made to plot the other mode in the same coordinate system. Steps taken along this line indicate a probability of failure which is related to the Fermi function. This may give some insight into the physical nature of the flaws. Since much is known about the behavior of the Fermi function, and since its use, in general, deals with minute contamination problems in high purity materials, perhaps it will prove useful in future preparation of fibers. Although the foregoing statement is highly speculative, it seems remarkable that Fermi-Dirac statistics should apply to glass fibers.

Since the publication of Progress Report 5, some effort has been made to evaluate the integral of equation 23:

$$F(\sigma, p) = \hat{n}_a^{-1} \int_0^{\sigma/K} \frac{\hat{n}_a!\, p^{n_a} (1-p)^{\hat{n}_a - n_a}\, dn_a}{(\hat{n}_a - n_a)!\, n_a!} \tag{23}$$

Several approximations have been attempted to reduce the integral to a closed form which have included Sterling's approximation and others. To date, none have been successful.

A trigonometic transformation has been performed on the integral, the result being

$$F(\sigma, p) = \sqrt{\frac{2\hat{n}_a}{\pi}} \int_0^{\cos^{-1}\left(\frac{\sigma_1 - 2^{\sigma}}{\sigma_1}\right)} \exp \hat{n}_a$$

$$\left[\cos^2 \phi \log\left(\frac{\cos^2\theta}{\cos 2\phi}\right) + \sin^2 \phi \log\left(\frac{\sin^2\theta}{\sin^2\phi}\right)\right] d\phi \tag{24}$$

where $p = \sin^2\theta$. By inspection it can be seen that computer limitations forbid a numerical solution directly. The original summation would be of this order of difficulty. It is implicit in the assumption that the summation will include numbers varying over an extreme range in magnitude.

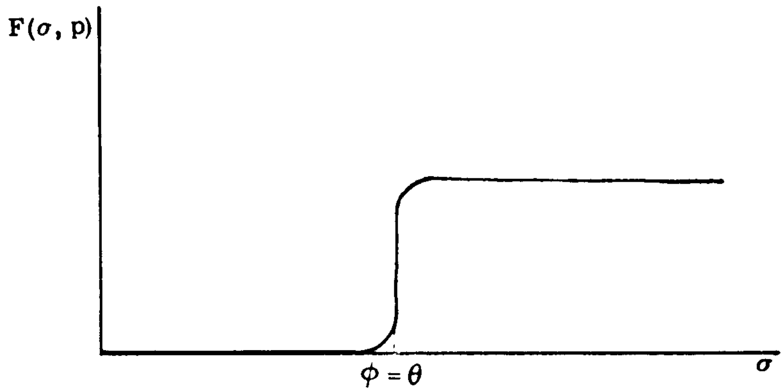

FIGURE D - 1. SHAPE OF F(σ, p)

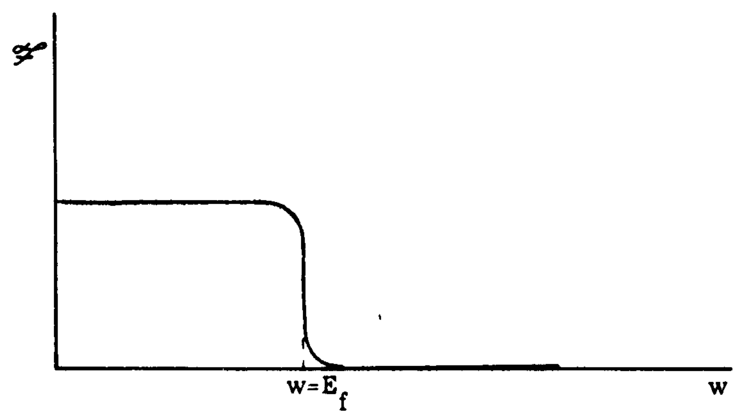

FIGURE D - 2. FERMI FUNCTION, $\mathcal{F}$

Looking again at the trigonometric transformation one can see that for $\phi = \theta$ the integrand goes to 1. Due to $\hat{n}_a$ being larger, it is obvious that the intergrand falls off to zero on both sides very rapidly, (i.e., for small changes in θ (or σ). Thus, upon integrating, the shape of the function is reminiscent of the Fermi distribution function, the distinction being shown graphically in Figures 1 and 2. In some form, then F (σ, p) should look like 1 - $\mathcal{F}$ where $\mathcal{F}$ is the Fermi function. Generally the Fermi function is expressed as

$$\mathcal{F} = \frac{1}{1 + \exp\left(\frac{W - E_f}{k\,T}\right)}$$

where W is the energy, E_f is the Fermi lend, T is the absolute temperature, and k is Boltzmann's constant. Fitting F (σ, p) with the appropriate parameters should lead to the desired result. Presently steps are being taken along this line.

It might be noticed that F (σ, p) is a probability of failure. This is the reason for the 1 - $\mathcal{F}$ comparison.

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1118 22nd Street, N. W.
Washington 7, D. C.

The Bendix Corporation
Bendix Products Division
South Bend 20, Indiana
Attn: Mr. Wade Hardy

Black, Sivalls and Bryson
Oklahoma City, Oklahoma
Attn: Mr. J. Carter

B. F. Goodrich Company
500 S. Main
Akron, Ohio
Attn: Mr. H. W. Stevenson

Goodyear Aircraft Corporation
Akron 15, Ohio
Attn: Mr. R. Burkley

Bureau of Naval Weapons Rep.
P.O. Box 504
Sunnyvale, California

Bureau of Naval Weapons Resident Rep.
P.O. Box 1947
Sacramento, California

Bureau of Naval Weapons Branch Rep.
Allegany Ballistics Laboratory
Cumberland, Maryland
Attn: Code 4

Bureau of Naval Weapons Resident Rep.
(Special Projects Office)
c/o Hercules Powder Company
Bacchus Works
Magna, Utah

Narmco Industries, Inc.
Research and Development Division
8125 Aero Drive
San Diego, California
Attn: Mr. W. Otto

Walter Kidde Company
Aerospace Division
Belleville, New Jersey
Attn: Mr. T. Siuta

General Electric Company
Schenectady, New York
Attn: Mr. T. Jordan

Hercules Powder Company
P.O. Box A
Rocky Hill, New Jersey
Attn: Mr. R. Carter

Rocketdyne Engineering
A Div. of North American Aviation, Inc.
6633 Canoga Avenue
Canoga Park, California
Attn: Mr. E. Hawkinson

Lockheed Missiles and Space Company
A Division of Lockheed Aircraft Corp.
3251 Hanover Street
Palo Alto, California
Attn: Mr. M. Steinberg

Commander
Aeronautical Systems Division
ASRCNC-1
AF Systems Command
U. S. Air Force
Wright-Patterson AFB, Ohio

Plastic Evaluation Center
Picatinny Arsenal
Dover, New Jersey
Attn: ORD-BB

Commander
U. S. Naval Ordnance Test Station
China Lake, California
Attn: Mr. S. Herzog
Code 5557